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FERROGRAPHY

BY

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SUMMARY

A new process is described which we have named "Ferrography." The process enables graphic information to be recorded on magnetic materials and to be reproduced on paper in visual form. Magnetic inks are available in black or colors, enabling monochrome or color reproductions to be made. The magnetic matrix may be used repeatedly so that multiple printing is possible, and the matrix may be stored indefinitely. The process has advantages: economy and speed of operation, permanence of image, and ability to record any information which is available in signal form, such as instrument data, facsimile transmission, or scanned images.

INTRODUCTION

As an outgrowth of studies in the reproduction of pictures, we have developed a general method of printing from electromagnetic records which we have named "Ferrography." Essentially, the method makes use of the magnetic-retentive properties of ferrous materials to record and retain an image pattern, and of the magnetic-attractive properties of other ferrous materials to assume the shape and opacity of the image, and in turn transfer it to a permanent support. Electrical means are required to receive the image and convert it into suitable signal form so that it may be magnetically recorded.

EXPERIMENTAL WORK

Since our purpose was to reproduce photographic images, we constructed a scanning device which held a photographic negative in a convenient manner (see Fig. 1). The recording cylinder was of transparent plastic and the reproducing cylinder of brass. A light source of small cross section was mounted together with a photocell on a lead screw in such a way as to provide continuous helical scanning. The

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signal from the photocell was fed into an amplifier, and used to modulate a carrier of suitable frequency. In order to achieve a record having the correct scale of tone values to reproduce the original picture, it was found necessary to warp the signal. In other words, compensating distortion is deliberately introduced to overcome certain deficiencies of the system and achieve the proper appearance of the final image.

The corrected signal is fed into a recording head similar to those used in audio magnetic recording (1).[2] It consists of a high-permeability core shaped as an all-but-closed ring. The gap is as narrow as practicable, consisting of a non-magnetic (brass) spacer 0.0005 in. thick.

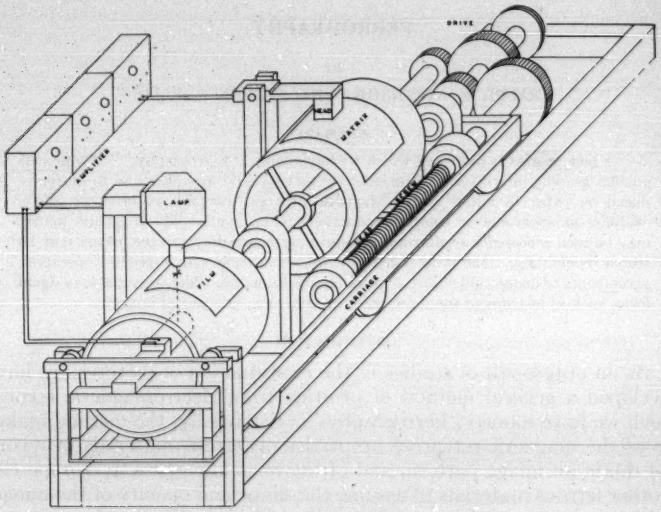

FIG. 1. Working model scanner and recorder.

The width of the head is as narrow as the width of the scanned line. We have done much work at 120 lines per inch using a head approximately 0.008 in. wide. In order to prevent excessive wear of the heads, they have been provided with bearing surfaces of synthetic ruby. The core is wound with a number of turns of fine wire, and the impedance of the head matched to that of the output of the amplifying circuit.

Experiments showed that iron-oxide coated film, of the type used commercially in magnetic sound recording, was a suitable medium for recording images. The range of magnetic field intensity available was sufficient to record a useful scale of tone values, the degree of resolution was finer than that of the scanning, and the material was readily avail-

[2] The boldface numbers in parentheses refer to the references appended to this paper.

able in various sizes and thicknesses so as to be a convenient practical medium. A sheet of this material was wrapped around the brass cylinder, and the head connected to the lead screw in such a manner that it passed over the film in an exact trace of the helical scan of the picture.

To keep dirt—the audio equivalent of noise—at a minimum, it was found desirable to erase the film just before recording. A suitable high-frequency bias was also found necessary. After balancing the circuit for the various factors involved, a rapid scanning of the original picture gives a magnetic record which we call a matrix. The matrix has a retained magnetic field varying in intensity according to the density of the original picture (see Fig. 2).

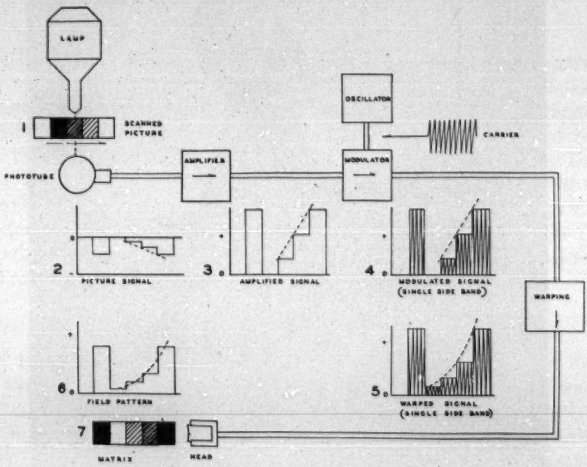

FIG. 2. Wave transmission of pictures.

This matrix may be handled, exposed to light, stored, and in all respects treated as the thin sheet of plastic which it is. On immersion in a suspension of ferro-magnetic material, the recorded image attracts the magnetic particles in the suspension, and immediately the pattern of the original is built up as a relief image on the surface of the matrix. After rinsing, the face of the matrix is pressed against a sheet of paper, and a clear sharp image of the original is reproduced. The matrix may be reinked and transferred repeatedly. It may also be erased and used again as a recording medium for a new picture.

Following the procedure outlined above, we have made accurate reproductions of both positive and negative black and white photographic originals. While there are imperfections, the results are suffi-

ciently good to testify to the usefulness of the method wherever its characteristics indicate it has economic or technological application. A ferrographic print is reproduced as Fig. 3.

FIG. 3. Experimental ferrographic prints.

We have found means of preparing inks with suitable magnetic properties having a wide range of color-absorption characteristics, and

we have achieved a three-color reproduction of a photographic transparency.

MATERIALS FOR FERROGRAPHY

Matrix

While sound recording film has been found convenient for the work, other materials are suitable also. It is necessary only that the record be made in a medium having a high magnetic retentivity and sufficient resolution for the purpose at hand. Tape or cylinders of steel having the desired magnetic properties would be useful. It is possible to use any shape or size of material that is convenient for handling and recording, provided only that the surface is coated with a layer of a magnetic-retentive material.

Ink

The basic requirement of the ink is that it consist of fine particles having magnetic attraction. Essentially, powdered iron particles will work, and we have used this material both dry and in liquid suspension for satisfactory reproductions. Its chief drawbacks are that it does not appear black against white paper, and it is readily oxidized by the air (rust) Fortunately, some of the oxides of iron work well without these drawbacks. The choice of liquid for suspension is more of a problem, and is determined both by the magnetic material used and by the paper and method of transfer used. We have used aqueous suspensions, as well as mixtures of other liquids.

Transfer

The particular method of transfer used is determined by the nature of the support and by the nature of the operation, as, whether slow or fast, single or multiple. Essentially, the transfer is mechanical; that is, the image on the matrix is composed of varying amounts of material, virtually all of which is physically transferred from the surface of the matrix to the surface of the support. Since it is held to the matrix by magnetic attraction, there must be some force exerted to remove it and attach it to the support. The simplest procedure is to have the image in the form of dry powder and apply it to a dampened paper support. The wet fibers of the paper attract and hold the powder. Another method uses gelatin- or glue-coated paper which is wet before the transfer. It is generally necessary to rinse the matrix between inking and transferring to remove surplus ink.

THEORETICAL CONSIDERATIONS

The use of Ferrography opens a new field for the recording and reproducing of visible images where the speed and energy requirements cannot be met by conventional methods.

The problem of the reproduction of visual information can be ex-

pressed as a transfer of energy in a three-dimensional array. From the standpoint of ordinary human vision, it is customary to think of the light pattern reflected or transmitted from a picture as a unit. It actually consists, however, of a large number of discrete elements, and any picture which appears to be a continuous array of tone values may likewise be broken up into a large number of discrete elements having discontinuous values. Where the whole pattern exists at one time, optical and photographic methods serve admirably to reproduce it. However, there are increasing numbers of applications where information is available in discrete units or in linear form, usually as an electrical signal, and it is desirable to record this information in a visual pattern. Examples are facsimile, television, and oscilloscope data. Photography and other methods have been pressed into service to record this information, but where transducers are required to convert electrical energy into light or mechanical energy, at very high speeds, or very high frequencies, there are limitations.

In all work with electrical signals we have only one variable, the strength of the signal, which may vary with time. If we must convert a three-dimensional array, as a picture, to electrical form, we can use conventional scanning methods to determine the two dimensions of position on the picture, and allow the variation in image density to motivate the signal variation, thus reproducing the array in three dimensions.

We record the signal variation as a variation in the magnetic field in our matrix, each element being given the intensity necessary to reproduce the image density of its corresponding element in the original. It is possible to use any suitable combination of devices to acquire and relay the pattern, and the final array may not be a picture in the conventional sense, nor need it have originated as a complete array. It is only necessary that a magnetic field of suitably varying intensity be recorded in a medium which may be inked and transferred.

CHARACTERISTICS

The ability of Ferrography to be useful in reproducing visual information requires that three factors be favorable: namely, the resolution of detail, the amount of energy required to produce a permanent change in the medium, and the time interval over which that energy is required to operate. Fortunately, all three requirements are met, and in such a way as to allow devices to be built which surpass existing methods of recording visual information. The following data obtained from our work indicate some of these characteristics.

Resolution

One of the primary considerations in all graphic reproduction is resolution. With Ferrography, we have found 100 lines per inch to be

the minimum for good results; 200 lines per inch produces good quality, and the linearity is scarcely visible; 400 lines per inch probably represents a maximum beyond which the gain in appearance and legibility is not worth the precision required.

Experience has shown that we get extremely sharp edges to the recorded lines. This leads to the fact that resolution profits from elements slightly shorter than their width—in other words, a vertical scan rate about 20 per cent greater than the horizontal rate. This requires circuits which will carry frequency components about 1.2 times the scanning pitch. For example, a scanning rate of 200 lines per inch will show improved appearance up to about 240 elements per inch along the line of scan. Beyond that, little improvement is noted.

Both inks and matrix materials appear to have resolving power well beyond the scanning definition.

Speed of Scanning and Recording

On our working model, we scanned and recorded satisfactorily on a 5-in. diameter drum, rotating at 1500 rpm. This gives 23,550 lineal inches of scan per minute, or 117 square inches per minute at 200 lines per inch. Using 120 lines per inch, the present facsimile standard, we recorded at the rate of 195 square inches per minute. We believe that with proper mechanical work, good recording could be obtained on a 12-in. drum rotating at 1000 rpm. At 120 lines per inch, this would do an area 16 in. by 20 in. in a little more than a minute.

Amplification Requirements

Because the impedance of the head changes with the frequency, a non-linear output is obtained. This dictates the use of a carrier frequency and wide band amplifiers. Experience showed that the carrier need only be about three times the highest picture modulating frequency. Single side bands are sufficient for good reproduction. Line copy can be reproduced without concern for the non-linear magnetic retentivity; however, continuous tone requires the use of a linear response. This we obtained by designing suitable semi-log circuits which give control of contrast and density, the two factors affecting tone-scale reproduction.

Power Requirements

Power requirements are of a low order, one watt being all that is needed. This simplifies the design of the amplifier; for example, we have found that an amplifier with a voltage gain of 500× with a single-ended power stage fills the requirements. The use of a photo-multiplier tube at less than maximum voltage reduces the need for noisy, high-gain amplifiers. Wave-form distortion of the carrier is not serious because only the total power present in a cycle affects the image.

Scale Reproducible

It is commonly regarded as satisfactory if a black and white photograph has a light to dark ratio of about 32 to 1, which corresponds to a density range of about 1.5. Using black magnetic ink we have been able to achieve a fairly satisfactory black, and the white area is very slightly affected in the transfer. We have been able to record over such a range that we believe Ferrography will have approximately the same ability to reproduce tones as photography. It has the advantage that the scale of reproduced values can be electrically controlled and corrected to the degree desired.

Limitations

There are two theoretical limitations to the efficiency of recording by means of Ferrography. First is the fact that the dimensional resolution is determined by mechanical limitations, and we have found that machine tolerances are the limiting factor in this respect. It is likely that resolution finer than 0.001 in. will be found to require excessively rigid mechanical work.

The second limitation is that known as the gap effect. In common with all methods of recording information, when the signal varies by an amount approaching the critical dimensions of the recording device, in this case the width of the non-magnetic gap in the head, no resolution is possible. In the case of a moving magnetic recording medium passing by a magnetic head, it is well known (1) that the number of changes in intensity which can be recorded in unit time is limited by the travel of the medium divided by the width of the gap. Since there are practical limitations to the maximum speed of the medium and to the minimum dimensions of the gap, there is an upper limit to the frequency of change which can be recorded.

PRACTICAL APPLICATIONS

In recording images which have been transmitted by wire or radio (the art of facsimile), many technics have been used, including mechanical, electro-chemical, and photographic transducers. There is an obvious advantage in a method using the energy directly in the electrical form, and Ferrography should also be useful in this art because of the speed with which information can be recorded and reproduced, in either single or multiple copies.

An important problem is the cost and quality of picture reproduction, especially in color. Because the problem of color printing involves the failure of dyes, both in the original and in the reproduction, to absorb light according to an ideal three-color system, much work has been done on methods of color correcting in printing (2). An excellent method (3) makes use of the feed-back principle in amplifier circuits, so

that it is necessary to scan the original and make the information available in signal form. The Ferrographic method of recording and reproducing this information presents many advantages.

Rather than supplanting existing processes, new methods usually find their most immediate application in some fields which have hitherto been unattainable, and it is in such a field that we expect Ferrography to develop most rapidly, namely, the recording of data from instruments. The requirements for speed and high frequency in this field are such as to make Ferrography unique in its ability to record and reproduce information in graphic form.

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(3) A. C. HARDY AND F. L. WURZBURG, JR., "Color Correction in Printing," *J. Optical Soc. Am.*, Vol. 38, pp. 300–307 (1948).

Shuttle Car for Mine Application.—A new cable-reel crawler-type steel-track shuttle car for mine application has been announced by the Locomotive and Car Equipment Department of the General Electric Company. The new unit, designed for operation in close quarters, has a turning radius of from 14 to 16 ft.

Radically different from previous shuttle-car designs using rubber-tired wheels, the new unit provides for light ground-loading pressure per square inch by distributing weight over a large ground area.

Because no clearance is required for turning wheels, a much wider conveyor is built into the body of the track-laying car for the same over-all width. Improved maneuverability is illustrated by the track-laying shuttle car's ability to turn by locking one track completely.

Because of a wider conveyor, unloading speed is increased. Simplicity is stressed by the use of only two drive motors, one on each track, eliminating complicated gears, drive shafts and steering mechanisms required by the conventional four-wheel-drive, four-wheel-steer shuttle car.

The new 250-volt car is driven by a two-sealed type, 20-hp., 250-volt series-wound motors driving each track through single gear reduction, chain and sprocket. The gear case is an integral part of the motor. One sealed-type, 3-hp., 250-volt compound-wound pump motor drives hydraulic auxiliaries. The conveyor chain is driven by two 5-hp., 1750-rpm. compound-wound gear motors, with a 44–1 gear ratio.

The cable reel is hydraulically driven with sealed-type enclosure. A spooling device controls level winding. The reel is equipped with 600 ft. of No. 3 two-conductor cable with ground wire, and guides permit reeling in either direction. The hydraulic motor torque is automatically compensated for reeling and unreeling cable pull.

All internal control elements, including accelerating and headlight resistors, are in sealed-type cases. Motor acceleration and reversing are accomplished by means of magnetic-type contactors actuated through time-delay relays by a foot button, providing 3-step starting. Brakes on both traction motors provide for steering.

Two steering levers at the operator's position operate the tracks. They are connected to the traction motor power circuits and to hydraulic brake valves. By moving a steering lever forward a few degrees, the corresponding traction motor is disconnected from the line, and the car turns slightly. Hydraulic pressure is applied to the steering brake by advancing the steering lever farther. For a very sharp turn, one track can be locked.

The track carriage is hinged to the body at the track drive end. The carriage is flexibly connected to the body on the front end by means of a torsion spring. The front main idler is positioned by a track recoil spring to absorb shock and maintain proper track tension.

The track linkage consists of forged hardened steel side-links with hardened steel pins and bushings. The track shoes are of hardened wear-resistant steel, smooth on the bottom. Each track runs on six rollers, and is supported over the top of the track carriage by two idler rollers.

RECENT PROGRESS IN THE PLASTIC METHODS OF STRUCTURAL ANALYSIS

BY

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PART I *

ABSTRACT

The "plastic methods" are concerned with the conditions of failure of statically indeterminate structures of ductile metal, mild steel in particular. Continuous beams and frames of mild steel can carry loads considerably in excess of those which cause the elastic limit of the material to be first reached. A real physical failure occurs when the load has reached a certain fairly well-defined value, above which small load increases rapidly produce much larger permanent deformations than had occurred at lower load levels. The plastic methods attempt to base the design upon this realistic concept of failure, and hence attempt to eliminate some of the uncertainty necessarily involved in design methods which do not deal directly with real structural failure.

This paper reviews recent developments in methods of plastic failure analysis based on certain simple hypotheses as to the carrying capacity of flexural members. The basic hypotheses are first defined and a simple example is given to illustrate their use. This serves also to explain the philosophy and advantages of the plastic methods in suitable applications. Some weaknesses of the basic hypotheses and limitations of the plastic methods are indicated. In treating general frames the need for a sufficient description of the manner of loading is pointed out. Two extreme types of loading termed "proportional loading" and "variable repeated loading" are defined; any actual loading program lies between these two extremes. The remainder of Part I is devoted to stating the mathematical theorems appropriate to the problem of proportional loading and to explaining techniques by means of which failure loads of this type may be readily computed for general continuous frames. A problem of a two-span portal frame is solved in order to illustrate the new techniques.

1. INTRODUCTION

Investigations concerned with developing the plastic methods of analysis and design of continuous frame structures have been proceeding in two general directions. In one, simple hypotheses are adopted concerning the behavior of mild steel structural members loaded beyond the elastic range, and effort is concentrated on developing methods for the analysis of frames which will be mathematically sound and complete, and as simple as possible while at the same time applicable to general types of structures and loadings. The objective is the development of simple and general methods for predicting the loads at which failure occurs in the definite sense contemplated in the theory. The second type of investigation consists of studying the detailed behavior of members, connections, and frames subjected to various definite loads.

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Its objective is the understanding of difficulties which result in certain cases from the adoption of simple hypotheses which are sometimes poor idealizations of the behavior of actual steel members.

Both types of investigation are necessary and complementary to each other. Obviously a complete and elegant theory based on simple hypotheses, by means of which theoretical failure loads could be readily predicted, would be of little value unless the special cases in which the simple hypotheses are inadequate are thoroughly investigated and understood. On the other hand, if effort is concentrated solely on exploring the difficulties and understanding in detail the highly complex behavior of members in the plastic range then it is possible that no methods would be developed which would often be approximate but yet simple and accurate enough for a wide range of practical use.

This review is concerned primarily with recent developments in methods of analysis of general frame structures. Certain physical assumptions are made concerning the carrying capacity of flexural members of mild steel. On the basis of these assumptions a self-consistent mathematical theory has recently been developed together with simple computational techniques which allow the ultimate loads on general continuous frame structures to be readily determined. The objectives of the first type of investigation may be said to have been achieved; having defined certain basic hypotheses a mathematical theory is based on them which is as rigorous and complete as the classical theory of the analysis of frames in the elastic range, based on the hypotheses of Hooke's law and continuity of deformations. Practical numerical analysis of frames of virtually any complexity can be carried out with a simplicity and rapidity which in most cases contrasts very favorably with that of elastic analyses for the same cases.

The underlying physical hypotheses are idealizations of actual properties which are rarely strictly correct and in some cases quite inaccurate. Hence difficulties arise in certain cases which invalidate the theory. These special problems are the subject of the second type of investigation mentioned above. While these important special problems will be emphasized in this review at appropriate places, and some references to recent work and indications of needed investigations will be given, no attempt will be made to discuss them adequately.

On balance, the experimental evidence now available seems to justify the point of view that methods of analysis based on the simple hypotheses will be suitable as the basis for practical applications, provided these methods can be corrected or refined for the special cases where the simple theory is inadequate. Design proposals can hardly be made until these cases are more fully understood. It is hoped that knowledge of the recent developments in methods of analysis based on simple hypotheses may serve to stimulate and to assist the investigation of these outstanding special problems.

2. BASIC HYPOTHESES

The relation of bending moment to curvature in each member of a continuous frame is the basic physical property concerned. The typical characteristic taken as the basis of the plastic methods is shown in Fig. 1. If the initially unstrained and unstressed member is bent (in pure bending) in a sense chosen as positive, the curvature first increases linearly in the usual elastic range. This is terminated when the bending moment reaches a value M_y. When the moment is increased above this value the curvature increases at a more rapid rate. The basic hypothesis is that as the bending moment approaches a value M_p characteristic of a given material and cross-section the curvature increases to indefinitely large values. The value M_p has been termed the "fully plastic moment" (1, 2)[3] or the "limit moment" of a member. It corresponds

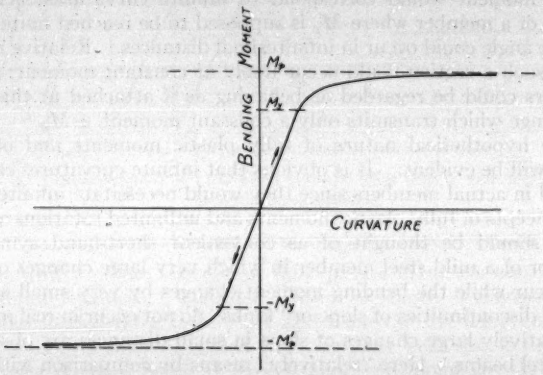

FIG. 1. Type of bending moment—curvature relation assumed.

to the spread of plastic zones over the whole cross-section except for a lamina of infinitesimal thickness in which the stress changes from the yield stress in tension to that in compression.

If the bending moment had been applied to the initially unstrained and unstressed member in the opposite sense to that taken as positive, a similar curve would be obtained; the elastic range would end at a value $-M_y'$, and the bending moment would tend asymptotically toward a fully plastic moment $-M_p'$ as the curvature tends toward negatively infinite values. For steel members in virtually all cases the values M_p and M_p' are equal, as will be evident when the calculation of the fully plastic moment is discussed. This equality of M_p and M_p' will be assumed in the following, for simplicity.

[3] The boldface numbers in parentheses refer to the references appended to Part I of this paper.

Mathematically, the plasticity conditions for each member of a frame are a pair of inequalities

$$-M_{pi} \leqslant M_i \leqslant M_{pi}, \tag{1}$$

where M_i is the bending moment at any section of the i-th member and M_{pi} is its fully plastic moment. The further condition is understood that increments of bending moment always have the same sign as increments of curvature, that is,

$$\frac{dM_i}{dk_i} \geqslant 0, \tag{2}$$

where k_i represents the curvature at a section where the moment is M_i.

The concept of "plastic hinge" formation follows directly from that of the existence of fully plastic moments. The attainment of a fully plastic moment would correspond to infinite curvatures; hence at a section of a member where M_p is supposed to be reached finite changes of slope angle could occur in infinitesimal distances. Relative rotations across such a section could occur freely at constant moment; thus the members could be regarded as behaving as if attached at this section by a hinge which transmits only a constant moment $\pm M_p$.[4]

The hypothetical nature of fully plastic moments and of plastic hinges will be evident. It is obvious that infinite curvatures cannot be realized in actual members since they would necessitate infinite strains. The concepts of fully plastic moments and unlimited rotations at plastic hinges should be thought of as convenient short-hand symbols for behavior of a mild steel member in which very large changes of curvature occur while the bending moment changes by very small amounts. Abrupt discontinuities of slope or "kinks" do not occur in real members, but relatively large changes of slope in small distances are observed in mild steel beams. Here "relatively" means by comparison with elastic behavior. The notions of limiting moments and plastic hinges are approximate but very useful representations of actual behavior.

To illustrate the observed behavior of real materials Fig. 2 shows some bending moment-deflection angle curves for mild steel wide-flange sections obtained by Luxion and Johnston (8). These were obtained with four-point symmetrical loading, and show that strains large compared with elastic strains are attained while the bending moment increases by hardly measurable amounts. A discussion of the calculation of fully plastic moments and of their dependence on various conditions is given later in Sections 4 and 5.

[4] According to Maier-Leibnitz (3) the concept of plastic hinges was first proposed by G. v. Kazinczy in the Hungarian journal *Betonszemle* in 1914. Various attempts to utilize the plastic properties of steel in the design of indeterminate structures were made in the 1920's. Bibliographies and discussions of this early work were given by F. Bleich (4) and by Maier-Leibnitz (5). Girkmann (6) was the first to discuss portal frames and more complex continuous frames. Van den Broek (7) states that the basic ideas of "limit design" were used as early as 1910 by C. M. Goodrich in the design of transmission towers.

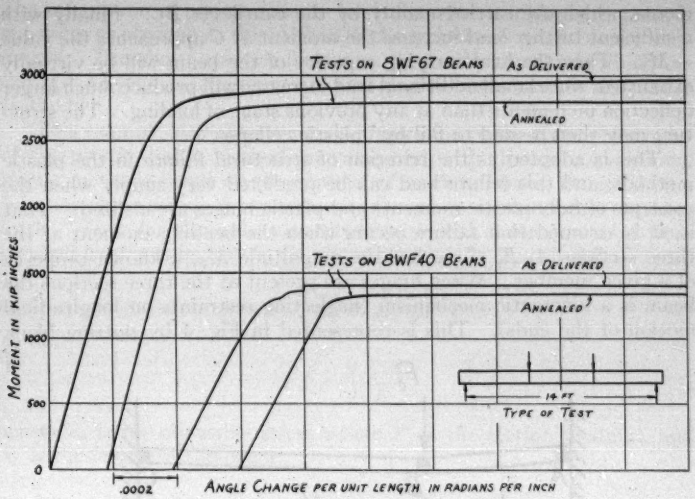

FIG. 2. Curves obtained by Luxion and Johnston (8) in tests of wide flange beams using third point loading.

3. OBJECTIVES AND LIMITATIONS OF PLASTIC DESIGN METHODS

The utility of the concept of fully plastic moments may be shown by a simple example. Consider a beam of length L clamped at both ends and loaded by a force P at $L/3$ from the left-hand end, Fig. 3. When P is gradually increased from zero the behavior is first elastic, the largest moment being at the left-hand end, point A. When P has

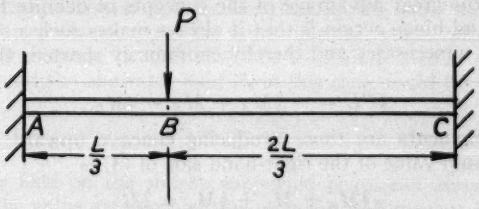

FIG. 3. Problem of beam with fixed ends.

been sufficiently increased yielding occurs at A, and the bending moment approaches the value $-M_p$. When the load is further increased the moments at B and C increase somewhat more rapidly than before, and that at B next approaches the value M_p. Beyond this load the segment AB contributes very little toward supporting further load incre-

ments, which are carried mainly by the cantilever BC. Finally with a sufficient further load increase the moment at C approaches the value $-M_p$. Then the load-carrying capacity of the beam will be virtually exhausted, since small additional load increases will produce much larger deflection increments than at any previous stage of loading. The structure may then be said to fail by "plastic collapse."

This is adopted as the criterion of structural failure in the plastic methods, and this failure load can be predicted very simply when the concepts of fully plastic moments and plastic hinges are utilized. That is, it is assumed that failure occurs when the bending moment at the three sections A, B, C reaches the magnitude M_p, a known property of a given member. When hinges are present at the three sections the beam is a kinematic mechanism (neglecting restraints on longitudinal motion of the ends). This is represented in Fig. 4 by putting black

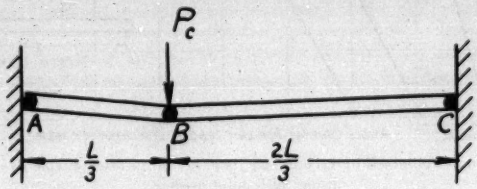

FIG. 4. Hypothetical collapse state; black circles represent plastic hinges.

circles at the hinge sections. The load which is in equilibrium with the moments at this stage is readily calculated and is a useful approximation to the observed failure load. One way to calculate it would be to follow the load up gradually and make a series of elastic-plastic calculations, starting with the beam as initially clamped, and taking account of the development of plastic zones at A, B and finally C. However, the great advantage of the concepts of definite fully plastic moments and hinge action is that it always makes such a step-by-step calculation unnecessary and thereby enormously shortens the analysis. By statics[5]

$$\tfrac{3}{8}PL = -\tfrac{3}{8}M_A + M_B - \tfrac{1}{8}M_C. \tag{3}$$

(Positive moments are those producing concave upward curvature.) The maximum value of the right-hand side of (3) is

$$\tfrac{3}{8}M_p + M_p + \tfrac{1}{8}M_p = 2M_p.$$

The maximum value of P is, therefore,

$$P_C = \frac{9M_p}{L}. \tag{4}$$

[5] Here (as throughout this paper) it is assumed that deflections are still small enough so that shape changes need not be taken into account.

The calculation of plastic collapse loads for more complex structures is not quite so obvious as this, although the techniques available are very simple. The above example illustrates the fundamental role which the concepts of fully plastic moments and plastic hinges play in the plastic methods.

This example also gives a simple illustration of the basic philosophy of the plastic design methods as compared with that of conventional elastic methods based on the concept of working stress, the maximum (elastic) stress which the member is allowed to sustain under working loads. Thus for a simply supported beam with a force at $L'/3$ from one end as shown in Fig. 5, the allowable load P_a' is such that

$$\sigma_a = \frac{M_{max}}{Z'} = \frac{2}{9}\frac{P_a'L'}{Z'},$$

so that

$$P_a' = \frac{9}{2}\frac{Z'\sigma_a}{L'}, \qquad (5)$$

where σ_a is the allowable stress value, Z' is the section modulus, and L' is the length of the beam.

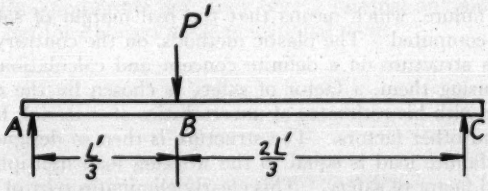

FIG. 5. Problem of simply supported beam.

If a fixed-ended beam is to be designed, of the same material and type and for a similar purpose, the same allowable stress value σ_a would probably be chosen. In this case the maximum elastic moment is $-4/27\,PL$, and the allowable load P_a in this case would be

$$P_a = \frac{27}{4}\frac{Z\sigma_a}{L}. \qquad (6)$$

Now the load on the simply supported beam can actually be increased to the value P_c' at which the moment at B reaches M_p', before failure in the sense of suddenly increasing deflections occurs; thus failure occurs at

$$P_c' = \frac{9}{2}\frac{M_p'}{L'}. \qquad (7)$$

In the case of the fixed-ended beam the failure load was shown above

to be

$$P_c = \frac{9M_p}{L}. \tag{8}$$

The actual margin of safety for the simply supported beam is, therefore,

$$\frac{P_c'}{P_a'} = \frac{M_p'}{Z'\sigma_a}. \tag{9}$$

On the other hand the margin of safety for the fixed-ended beam is

$$\frac{P_c}{P_a} = \frac{4}{3}\frac{M_p}{Z\sigma_a}. \tag{10}$$

Now if the beams are both of mild steel and of similar type (for example, both American standard I-sections) then the quantities $M_p'/Z'\sigma_a$ and $M_p/Z\sigma_a$ will be roughly equal. Then Eqs. 9 and 10 show that the fixed-ended beam will have been designed with a real margin of safety about 4/3 that of the simply supported beam. As Baker (1) has forcefully pointed out, this is indefensible. This difficulty is basic in elastic methods. No matter how rational it is otherwise, an elastic design based on an allowable stress value is incapable of dealing with structural failure, which means that the real margin of safety is not defined or computed. The plastic methods, on the contrary, base the design of a structure on a definite concept and calculation of failure load. In using them, a factor of safety is chosen by the designer in accordance with his estimates of uncertainties of material, fabrication, loading, and other factors. The structure is then so designed that its computed failure load is equal to the working load multiplied by the chosen load factor of safety. This clearly eliminates part of the uncertainty involved in a design by elastic methods.

Apart from considerations of the real safety of structures, elastic methods as conventionally employed suffer from the further disadvantage of basing designs on largely fictitious maximum stress values. The real maximum stresses in actual structures commonly have little relation to the maximum stresses computed in standard methods, even when these are "rational" rather than mere empirical formulas, because account is not normally taken of stress concentrations at holes, joints, corners, and the like. Moreover, the nominal elastic stresses in members are based on assumptions of end-fixity, continuity, freedom from initial misfits, etc. These conditions rarely exist in the actual structure, and unfortunately elastic stresses may be highly sensitive to variations in these conditions. The inability of conventional analyses to deal with these factors is well known, and allowance for them is commonly made by requiring the steel to have sufficient ductility. The plastic methods are less sensitive to unknown quantities such as initial stresses, movement of supports, etc.; for example, the failure load computed for the

problem indicated in Fig. 3 is independent (within wide limits) of initial stresses. In general, the plastic methods attempt to take account of the ductility of the material in as rational a manner as consistent with workable simplicity.

It should be emphasized that the plastic methods are by no means always appropriate. For example, possible failure under repeated loadings of the usual fatigue type, in which all members are nominally in the elastic range, cannot be dealt with by the plastic methods. Other important cases in which the plastic methods will probably not be applicable, except perhaps as an adjunct to other methods, are those in which specifications on permissible deflections govern the design. The plastic methods are based on a definite concept of failure involving the rapid *growth* of plastic deformation (per unit load increase). The deflections in a frame at the critical failure load can be estimated, as will be discussed in Part II of this paper. However, when the numerical values of deflections are of primary importance, rather than the ability of a frame merely to carry load without suffering sudden increases of distortions, then the plastic methods by themselves will generally be inappropriate.

When in the designer's opinion the criterion of failure in the plastic methods is the appropriate one, they offer a rational and simple basis for design,[6] and they always have educational value.

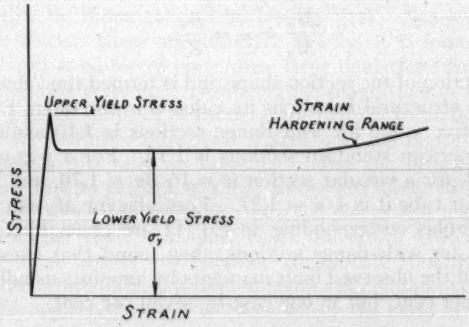

FIG. 6. Typical stress-strain diagram for mild steel.

4. COMPUTATION OF FULLY PLASTIC MOMENTS

The typical stress-strain relation of mild steel has the shape shown in Fig. 6. The lower yield stress σ_y is consistently measurable without great difficulty, while the upper yield stress is very sensitive to test conditions, especially speed of loading and axiality of test specimen; it may not appear at all if insufficient care to ensure axiality or eliminate

[6] Baker (19) has described important applications in the design of wartime structures, which were made in England during the late war.

residual stresses is taken. Morrison (9) has shown that the lower yield stresses in tension and compression are equal for mild steel. As already stated the fully plastic moment corresponds to the condition when plastic zones have spread over the whole section except for an infinitesimal transition zone at the neutral axis. In computing the fully plastic moment both strain hardening and the upper yield stress are safely neglected. Then the fully plastic moment can be computed as

$$M_p = Z_p \sigma_y, \tag{11}$$

where

σ_y = lower yield stress, and
Z_p = "plastic section modulus" = sum of moments of areas on both sides of neutral axis with respect to that axis.

The neutral axis is the axis (transverse to the plane of loading) which divides the cross-section into two equal areas. In the preceding discussion it is assumed that the loading is applied in a plane of symmetry of the section.

The "yield moment" M_y above which elastic behavior ceases can be computed as

$$M_y = Z \sigma_y \tag{12}$$

if the upper yield stress is ignored. Here Z is the usual section modulus of elastic theory. The ratio of M_p to M_y is

$$\frac{M_p}{M_y} = \alpha = \frac{Z_p}{Z}. \tag{13}$$

This is a function of the section shape and is termed the "shape factor." For common structural I-sections its value is in the range 1.05 to 1.25; a representative value for wide-flange sections is 1.10 while a typical value for American standard sections is 1.15. For a rectangular section α is 3/2, for a circular section it is $16/3\pi = 1.70$, and for a thin-walled circular tube it is $4/\pi = 1.27$. Formulas for M_p can be derived easily. Formulas corresponding to Eq. 11 are given by Luxion and Johnston (8) for wide-flange sections; they found that these formulas overestimated the observed limit moments by amounts usually less than about three per cent, but in one case by seven per cent.

5. VARIATION OF FULLY PLASTIC MOMENTS

The fully plastic moment is assumed in the plastic methods to be a known property of each member of a frame, which can either be calculated using Eq. 11 from knowledge of the material and the section dimensions or determined by a test of the member in pure bending. The possible inaccuracies in this assumption must be recognized. In the first place the lower yield stress on which the calculation depends is subject to considerable variation (as compared, for example, with Young's modulus). σ_y depends particularly on the heat treatment, the

chemical composition of the steel, and the rate of loading. The last two effects will probably be unimportant for most structural applications involving essentially static loading. The sensitivity of σ_y to heat treatment is shown up by the differences commonly observed (8) between values obtained for specimens cut from different parts of the flange and web of I-sections, as well as by tests such as shown in Fig. 2. Attention will have to be paid by designers to the specification of tolerances for yield stresses. This is only to be expected in a theory which aims to take quantitative advantage of the plastic properties of steel.

More serious variations than these are caused by differences in stress distribution. The limit moment of a member subjected to a transverse shear force or to an axial force is never identical with the fully plastic moment computed from Eq. 11 which applies strictly to pure bending. Moreover, adjacent joints, corners, section changes, or concentrated loading affect the actual value of M_p. Thus, for example, the value of fully plastic moment deduced from a test on a centrally loaded simply supported beam may differ by an appreciable amount (depending on beam dimensions) from that deduced from a test in which the beam is loaded mainly in pure bending, as shown by Roderick and Phillips (10).

Both shear and axial forces tend to lower the fully plastic moment from the value for pure bending. Estimates of both effects can be made analytically, and some experimental evidence has also been obtained, notably by Baker (1) and by Hendry (11). Space is not available here to discuss these adequately. Briefly, it is found that both shear forces and axial forces may cause large deviations from the limit moment for simple bending. However, the magnitude of these effects depends for a given shape of beam upon the ratio of beam depth to span length, and for ordinary structural shapes and the depth-span length ratios commonly met with in building frames the errors in value of limit moment due to neglect of shear forces are likely to be less than 5 per cent and those due to neglect of axial forces will usually be less than about 15 per cent. In many cases the errors will be much less than these; these figures are intended to indicate roughly the upper limits on these errors to be expected in frames of common types. Shear forces can be neglected except in cases where the ratio of span length to beam depth is less than about four to one, which is much smaller than commonly used. The effect of an axial force on the fully plastic moment can be neglected when the force is less than about a quarter of the compressive yield force of the member (1). This means, for example, that a portal frame carrying a central concentrated load on its horizontal member should have the length-depth ratio of this member greater than about 8, if cross-sections of all members are the same.

In the estimate of the effect of axial forces indicated above no account was taken of the influence of the deflections of a member on

the bending moment in it and thus on its load carrying capacity. The problem of the behavior of a strut subjected to an axial compressive force, and to end moments and shears is perhaps the most important special problem that has not yet been fully investigated. Contributions have been made by Chwalla (12) and by Baker, Horne, and Roderick (13), and investigations by the latter group are continuing.

6. PROBLEM TYPES

The plastic methods of analysis and design cannot take advantage of the principle of superposition of stresses and deflections due to individual loads, which is valid while the structure remains within the elastic range. This means that in typical problems of structures on which several loads are applied, as for example the two-span portal frame of Fig. 7, the manner in which the various loads H, P, Q are to be applied must be described fully enough so that the problem of plastic analysis is unambiguous.

This necessity for defining the manner of loading might seem to be a fundamental weakness of the plastic methods as compared with elastic

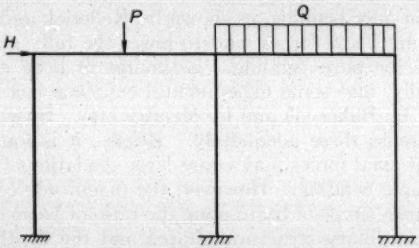

FIG. 7. Typical continuous frame and loading.

ones, since it is rarely possible in practice to predict or specify how a set of loads will be applied in relation to each other. However, if the extreme values (upper and lower limits) of each of a set of loads can be specified then two extreme loading conditions can be considered. In one, all the loads are imagined to be held in constant ratio to each other and gradually increased from zero to their final values. This is termed "proportional loading." In the other extreme loading condition no relation between the values of the loads at any instant is assumed, and also each one of the loads is supposed to be applicable to either its upper or lower extreme value an arbitrary number of times. This may be termed "variable repeated loading." It includes proportional loading by any combination of the loads as a special case.

Any program of loading obviously lies somewhere between the two extremes just defined. Therefore, methods of analysis to determine the

plastic failure loads appropriate to both types have been developed and will be discussed in this paper. As will be seen, failure under repeated variable loads is more difficult to analyze than failure under a single load application, primarily because elastic analyses for each of the individual loads must be made in the former case, while no elastic considerations are necessary in the latter. (The illustrative problem already treated of the beam shown in Fig. 3, is a simple example of proportional loading.) The main purpose of developing methods for handling variable repeated loads is to obtain indications of how much the failure load limits for this case will differ from those for proportional loading. Variable repeated loading will be discussed in Part II of this paper.

7. BASIC THEOREMS OF PLASTIC COLLAPSE UNDER PROPORTIONAL LOADING

When the loads on a frame such as that shown in Fig. 7 are gradually increased, remaining in fixed ratios to each other, then according to the concepts of the plastic theory the fully plastic moment is reached in turn at successive sections. At each such section a plastic hinge is assumed to form, at which rotations may then occur under constant moment. As each plastic hinge forms, the degree of redundancy of the frame is reduced by one. Eventually, at a certain load, plastic hinges will have formed at sufficient sections to transform the frame or some part of it into a kinematic mechanism. According to the concepts of the plastic methods the load cannot be increased above this value, and since deflections of arbitrary magnitude could occur the frame is said to fail by "plastic collapse." In practice, the growth of large deflections is halted mainly by strain hardening or by effects of shape changes and there is no sudden loss of equilibrium. What does happen is that small increments of loads begin to produce much larger increments of deflections than at lower load levels. Some further indications of observed behavior in tests will be given in Part II.

The direct analysis for the collapse load of general frames can be based upon two fundamental theorems which were recently established by Greenberg and Prager (14), following a suggestion of Feinberg (15).

The first theorem may be called the *statical principle*. Let W_c stand for the actual collapse load of a given frame (the ratios between all individual loads are fixed so that the complete load is specified by a single quantity). Now suppose W' is a load at which it is possible to find a complete system of bending moments which satisfy all equilibrium requirements and are such that the fully plastic moment of no member is exceeded. Then it can be shown that

$$W' \leqslant W_c. \tag{14}$$

In other words, the actual collapse load is the largest load at which it is possible to find a system of bending moments satisfying all equilibrium

conditions with that load and nowhere violating a plasticity condition.

The proof of this theorem is very simple, but full details will be omitted here. Greenberg and Prager proved first that at the collapse load W_c only plastic deformations, that is rotations at plastic hinges, occurred, all bending moments remaining constant. It is imagined that there exists a system of bending moments such that the fully plastic moment of no member is exceeded and such that equilibrium is satisfied with a load γW_c, where γ is a number greater than unity. This assumption is shown by a simple application of the principle of virtual work to lead to a contradiction, and thus the theorem is proved.

The second theorem may be termed the *kinematic principle*. Suppose plastic hinges are inserted at certain sections so that either the whole structure or some part of it is converted into a kinematic mechanism. Small deflections of arbitrary magnitudes could then take place under constant load, which would do positive work during the motion. For illustration, the frame of Fig. 7 might be made into a mechanism by putting hinges at top and bottom of the three legs of the frame, as shown in Fig. 8. The frame's position can be specified by a single

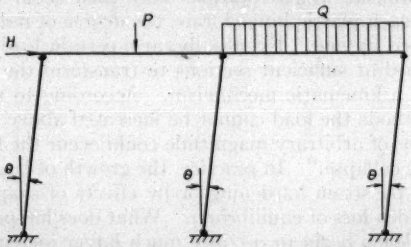

FIG. 8. Typical mechanism due to formation of plastic hinges.

quantity such as the angle θ, which can have any small value. (The assumption of smallness is made so that effects of changes of shape of the frame can be disregarded.) To any such mechanism there corresponds a definite load value, which could be found, for example, by applying the principle of virtual work, or directly from equilibrium considerations. Denote by W''' the load corresponding to any configuration of hinges which reduces the frame to a mechanism. Then according to the kinematic principle

$$W_c \leqslant W'''. \tag{15}$$

In other words, the actual collapse load never exceeds the load corresponding to any mechanism into which the frame is converted by a suitable disposition of plastic hinges.

An analytic proof of this theorem can be given which follows somewhat similar steps to those indicated for the statical principle. How-

ever, the following physical argument is attractive for its simplicity. Suppose a wrong choice of hinge locations is made. This must mean that the fully plastic moment in some member or members is exceeded, when moments at all sections are in equilibrium with the load corresponding to the chosen mechanism. Now imagine that at all locations where the fully plastic moment is exceeded the member is sufficiently strengthened, by increasing the yield stress of the material at those sections, so that in the modified structure all plasticity conditions are satisfied. The computed load would then be the correct collapse load for the hypothetical strengthened structure. This follows from the statical principle: if all plasticity conditions are satisfied and the structure is a kinematic mechanism, then the load with which the moments are in equilibrium is the actual collapse load of the frame. Now the effect of strengthening any part of a structure must be either to increase its collapse load or leave it unchanged. Hence the computed load, if it is wrong, must exceed the true collapse load of the given frame.

The continued inequality

$$W' \leqslant W_e \leqslant W'' \tag{16}$$

which expresses the two principles shows that the collapse load is *unique:* one and only one load value exists such that, with all bending moments satisfying statical equilibrium and nowhere exceeding a fully plastic moment magnitude, plastic hinges exist at sufficient sections to convert the frame into a kinematic mechanism.

There is, of course, no need to prove the uniqueness of the physical collapse load of a given frame. However, it is not self-evident that one could not find *on paper* two mechanisms with moments in equilibrium with two different loads and nowhere violating a plasticity condition. This possibility is ruled out by the uniqueness theorem stated. A direct proof of the uniqueness theorem was given independently by Horne (16), with proofs of the other basic theorems.

8. CALCULATION OF THE PLASTIC COLLAPSE LOAD

Baker (1, 2) first emphasized that the plastic failure load under proportional loading could be calculated directly, without the need for computations in the elastic or elastic-plastic range. Previous investigators, for example Maier-Leibnitz (5) and Girkmann (6), had made other important contributions, but had determined the plastic failure load by a series of elastic analyses, calculating the series of load increments required to produce each successive plastic hinge and stopping when a sufficient number of hinges had been formed to convert the frame or a part of it into a mechanism. This point is always indicated by the fact that for higher loads the equilibrium equations cannot be satisfied without violating the plasticity conditions of at least one member. Baker's direct method, which he applied to simple portal

frames and pitched roof portals, was to guess at a probable mode of failure, corresponding to the presence of plastic hinges at certain sections such that the frame became a mechanism. He then investigated by means of equilibrium equations or moment diagrams for the frame whether or not values of the remaining moments throughout the frame could be found which would not exceed the fully plastic moment of any member. If this was found to be the case the tentative mode was known to be the correct one, the uniqueness of the solution being intuitively recognized.

This method worked well for the frames mentioned, especially when the number of hinges in the failure mode was $r + 1$, where r is the number of redundant moments in the frame. In this case all of the moments are determined at once by simple statics. However, in cases of more complex frames the number of hinges is generally less than $r + 1$. Then the labor involved in investigating whether or not a complete moment system can be found satisfying both equilibrium and plasticity conditions becomes considerably greater. Although this examination could be done fairly easily for portal frames there was no guiding principle for examining failure modes in more complex frames. Moreover, once a tentative mode was shown to be wrong, in more complex cases it was not obvious how to proceed to the next trial in an advantageous way.[7]

Greenberg and Prager (14) suggested that complex frames be treated approximately by using the statical and kinematic principles to determine lower and upper bounds, respectively, on the actual collapse load. They showed how any upper bound could be made to determine a lower bound, after the complete moment system corresponding to any assumed mechanism had been determined. However, although their work greatly clarified the theoretical concepts, the computational difficulties involved in determining good lower bounds for complex frames were often much greater than desirable. Moreover there was still no rule for proceeding from one trial to the next.

A procedure which could be made completely automatic was presented by the present authors (17). This involved writing the pair of inequalities (1) for each member, combining these with the equilibrium equations relating the bending moments, and then systematically reducing them so as to evaluate the largest value of the load for which all of the plasticity conditions could be satisfied. However, unless this method could be adapted to high speed automatic

[7] The statement that $r + 1$ plastic hinges occur in the failure mode has been made frequently, but it is not true, as far as general frames are concerned. For example, the mechanism shown in Fig. 8 has $6 = r$ hinges. Under a suitable load system collapse of this frame might occur with as few as three hinges; this would be a "local" collapse involving one beam only, but is nevertheless a true failure of the sort considered here. On the other hand, examples can easily be constructed of cases in which several hinges form simultaneously at the failure load, so that the final number is larger than $r + 1$.

computing devices it would be too long, for complex frames, to be of practical use. Another systematic procedure was recently proposed by Heyman and Nachbar,[8] which also seems rather laborious. In this method upper and lower bounds on the failure load are made to approach each other until the gap is considered small enough.

The best practical procedure seems to be one which is not mathematically systematic but which follows well-defined guiding rules, involves very simple computations, and leads finally to the actual collapse load and mode of failure (18).

To explain this method we take as an illustrative problem the two-span portal frame with loads as shown in Fig. 9. The loads are a side load W, a concentrated force $2W$ on the left-hand beam and a distributed load $4W$ on the right-hand beam. The length of both hori-

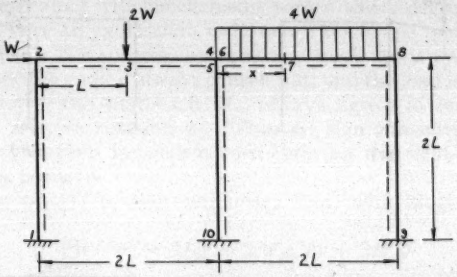

FIG. 9. Details of loading and frame used for illustrative example of plastic collapse analysis under proportional loading. Numbered sections are those where moments must be considered. Dashed lines indicate moment sign convention.

zontal and vertical members is taken as $2L$. The fully plastic moment of the legs is assumed to be M_p, while that of the horizontal beams is $2M_p$. When a sign convention for the bending moments in the frame is needed, positive moments will be taken as those which put tensile stresses in the side of a member adjacent to the dashed line drawn in Fig. 9.

The problem is to find the distribution of plastic hinges reducing the frame or some part of it to a kinematic mechanism, which requires no fully plastic moment in the frame to be exceeded. By the kinematic principle this corresponds to the *lowest* load of any possible mechanism, and this is the key fact in the method.

It must first be noted that plastic hinges can occur only under loads or at joints. One could not, for example, have an isolated hinge in the

[8] This approach was described in a paper "Approximate Methods in the Limit Design of Structures," presented to the First U. S. National Congress of Applied Mechanics, Chicago, Ill., June, 1951, and issued as Technical Report A11–54 of Brown University to Office of Naval Research, Providence, R. I., November, 1950.

middle of the left-hand vertical, because the shear force is constant in this member and therefore the moment varies linearly along the member. As far as the distributed load is concerned a hinge might occur anywhere along the length of the member it acts on. There are in this problem ten sections where plastic hinges can occur, and these are numbered in Fig. 9. The possible hinge location under the distributed load is considered to be at an arbitrary distance x from the left-hand end of this beam. Only moments at these ten sections need be considered, in investigating possible modes of failure.

Three particularly simple mechanisms are shown in Fig. 10(*a*), (*b*), (*c*). Of these (*a*) and (*b*) each involve one beam, while (*c*) involves only hinges at joints, but not under loads. These are termed the

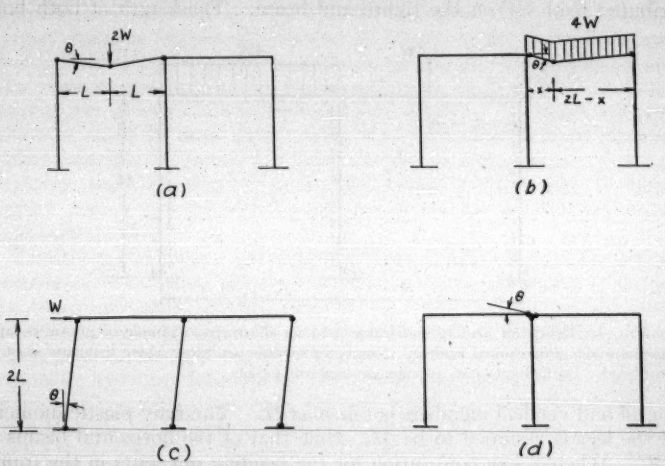

FIG. 10. (*a*), (*b*) and (*c*) are "elementary mechanisms" for frame and loading considered. (*d*) is fictitious mechanism representing small rotation of joint where three (or more) members are connected.

"*elementary mechanisms*" of the problem. Similar elementary mechanisms of these two types can always be distinguished in any frame. They will be termed, respectively:

(*A*) *Beam mechanisms:* each involving hinges at the end sections of one member and at one interior section. If more than one load acts on a given member an elementary mechanism can be defined for each load, with a hinge under that load. If no load is applied to a member no elementary mechanism exists for that member.

(*B*) *Frame or panel mechanisms:* each involving hinges at joints but not under loads on members, and implying general motion of a frame (or panel or story of a frame).

In the following any mode will be designated directly by its figure number.

The collapse load corresponding to each of these elementary mechanisms will now be calculated using the principle of virtual work. This principle takes the following form when it is applied to the small motion of a frame which is rigid except at joints where constant moments M_{pi} act and rotations ψ_i occur:

$$\sum_j P_j \delta_j = \sum_i M_{pi} \psi_i. \tag{17}$$

The left-hand side gives the work done by the external loads P_j on the small displacements δ_j which occur in a small mechanism motion. The right-hand side states the "internal work" done by the fully plastic moments M_{pi} on the small relative rotations ψ_i at the various hinges. Since we are dealing with mechanisms with a single degree of freedom the δ_j and ψ_i are all expressible in terms of the single parameter, such as the angle θ, which specifies an arbitrary small motion of the mechanism. Hence the angle θ always cancels out, and the load is determined. It will be noted also that each M_{pi} always agrees in sense with the appropriate relative rotation ψ_i. Thus no sign convention on either moments or rotations is needed, each term on the right-hand side of Eq. 17 being positive.

Applying Eq. 17 to the mechanism 10(a) with hinges at sections 2, 3, 4,

$$2W(L\theta) = M_p\theta + 2M_p(2\theta) + 2M_p\theta \tag{18}$$

$$WL = 3.5\, M_p. \tag{19}$$

It is to be noted that the hinge at section 2 occurs in the vertical member and the fully plastic moment there is M_p rather than $2M_p$.

In the case of the mechanism 10(b), with the hinge taken at an arbitrary distance x from the left-hand end, the virtual work equation yields

$$4W\left(\frac{\theta x}{2}\right) = 2M_p\theta + 2M_p\left(\frac{2L\theta}{2L - x}\right) + M_p\left(\frac{x\theta}{2L - x}\right), \tag{20}$$

$$WL = \frac{(8L - x)L}{2x(2L - x)}\, M_p. \tag{21}$$

Finally the mechanism 10(c) yields

$$W(2L\theta) = 6M_p\theta, \tag{22}$$

$$WL = 3M_p. \tag{23}$$

The reason for singling out these mechanisms for special attention is that the actual failure mode of the frame is either one of these or can be obtained as some *combination* of them. The correct mode is the one which requires the lowest load value of all possible modes. Hence, in combining mechanisms the objective is always to reduce the load, and

the virtual work equation shows that to reduce the load the work done by the load must be made as large as possible while the internal work of the plastic moments is made as small as possible. The following guiding rules can be stated:

1. Modes should be combined in such a way that a plastic hinge originally appearing in the two modes is eliminated. Only in this case can the failure load of the combination be less than that of either of the two modes being combined.

2. A joint where three or more members are connected should be rotated, if necessary, in order to reduce the internal work at this joint. It is helpful to think of a rotation of each such joint as motion of an additional "elementary mechanism," as indicated in Fig. 10(*d*), even though such a mechanism may be fictitious since either an equilibrium or a plasticity condition might be violated.

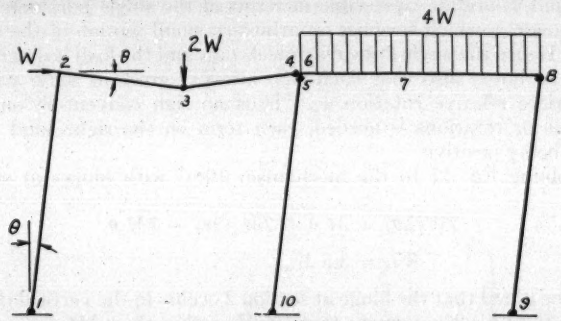

FIG. 11. Mechanism obtained by combining that in Fig. 10(*a*) with that in 10(*c*) so as to eliminate plastic hinge at section 2.

3. In combining a mode involving a hinge under a distributed load this hinge should first be assumed arbitrarily to be at the midpoint of the beam. If the actual mode appears to include this hinge, the correct position of the hinge under the distributed load can then be determined. In practice the change in failure load due to this adjustment is usually small (one or two per cent) and hence negligible.

Applying these rules to the present problem, it is first noted that the mode 10(*c*) is most likely to be correct of the elementary mechanisms, since its failure load is the smallest. This mode can be modified by combining 10(*a*) with it, choosing the angle θ equal in each case so that the hinge at section 2 is eliminated. The resulting mode is shown in Fig. 11. The virtual work equation for mode 10(*c*), Eq. 22, is then modified as follows:

$$W(2L\theta) + 2W(L\theta) = 6M_p\theta - M_p\theta + 2M_p(2\theta) + 2M_p\theta, \quad (24)$$

$$WL = \tfrac{11}{4} M_p = 2.75\, M_p. \quad (25)$$

This result could have been obtained directly from the new mechanism, Fig. 11, but it is often simplest to proceed by modifying a previous virtual work equation, as done in Eq. 24.

This combination led to a lower collapse load, so is more likely to be correct than any of the elementary modes. To find whether it is correct it is necessary to check the remaining combinations. Neither 10(*b*) nor 10(*d*) separately combines with 11 advantageously, since no hinge can be removed by such a combination. However, both these modes together can be combined with 11 so as to eliminate the hinges at sections 5 and 6. The new combination may or may not be advantageous, depending on the magnitude of the distributed load. The new

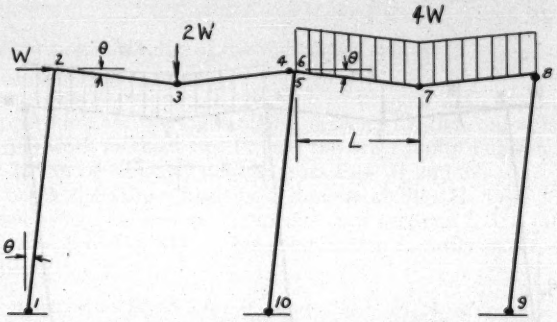

FIG. 12. Mechanism obtained by combining mechanisms in Fig. 10(*b*) and (*d*) with that in Fig. 11 so as to eliminate plastic hinges at sections 5 and 6.

mode is shown in Fig. 12. The virtual work equation for this mechanism is

$$4WL\theta + 4W\left(\frac{L\theta}{2}\right) = M_p(5\theta) + 2M_p(6\theta), \tag{62}$$

$$WL = 17/6\, M_p = 2.83\, M_p. \tag{27}$$

This result is approximate since, following rule 3, the hinge under the distributed load was arbitrarily assumed to be at the mid-point of the beam. If this hinge is put at x from the left end, as shown in Fig. 13, Eq. 26 becomes

$$4WL\theta + 4W\left(\frac{x\theta}{2}\right) = 12M_p\theta + 2M_p\left(\frac{2L\theta}{2L-x}\right) + M_p\left(\frac{x\theta}{2L-x}\right), \tag{28}$$

$$W = \frac{12 + \dfrac{4L+x}{2L-x}}{4L+2x} M_p. \tag{29}$$

The best value of x is that which minimizes W as given by Eq. 29. This value is found to be $0.982L$ and the corresponding minimum value of WL is still $2.83\ M_p$, the previous value being unchanged to three significant figures. In practice, such a determination of x would be required only if the load computed with the hinge at the mid-point was the lowest load or within, say, two per cent of the lowest load. Even then a few trials of x in the equation corresponding to Eq. 29 will probably suffice. Inspection of the bending moment diagram will indicate in which half of the beam the hinge lies.

All combinations of elementary mechanisms have now been tried. The one yielding the smallest value of W is shown in Fig. 11. This is therefore the correct collapse mode and the correct failure load is given by Eq. 25.

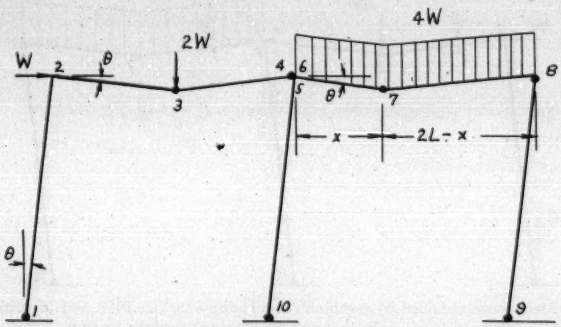

FIG. 13. Same as Fig. 12 but with hinge under distributed load at arbitrary location.

As an exercise the reader may verify that if the distributed load is $5W$ but the other loads are unchanged the true failure load is $2.61\ M_p/L$, with the collapse mode as in Fig. 12; while if the distributed load is $8W$ the failure load is $1.75\ M_p/L$, the local collapse mode of Fig. 10(b) then being the correct one.

9. RELATION OF EQUILIBRIUM EQUATIONS TO METHOD PROPOSED

An alternative method of calculating the failure load corresponding to any chosen mechanism is by direct use of the equations of equilibrium and the plasticity conditions, as already done for the simple example of Fig. 3, Section 3. Any mode of collapse corresponds to the particular equation of equilibrium which involves only the bending moments at the sections where hinges appear in the mechanism. Thus, corresponding to the elementary mechanisms of Fig. 10 there are four independent equations of equilibrium. These are the following, each

corresponding to the mode shown in the figure number listed,

$$\text{Fig. } 10(a): \quad M_3 = \tfrac{1}{2} M_2 + \tfrac{1}{2} M_4 + WL, \tag{30}$$

$$\text{Fig. } 10(b): \quad M_7 = \frac{2L - x}{2L} M_6 + \frac{x}{2L} M_8 + \frac{x(2L - x)}{L} W, \tag{31}$$

$$\text{Fig. } 10(c): \quad 2LW = M_2 - M_1 + M_5 - M_{10} + M_9 - M_8, \tag{32}$$

$$\text{Fig. } 10(d): \quad M_4 + M_5 - M_6 = 0. \tag{33}$$

Each of these equations that contains the load W determines an upper limit on W when account is taken of the plasticity conditions on the bending moments. For example, Eq. 30 can be rewritten as

$$WL = M_3 - \tfrac{1}{2} M_2 - \tfrac{1}{2} M_4. \tag{34}$$

The largest possible value of the right-hand side is

$$2M_p - \tfrac{1}{2}(-M_p) - \tfrac{1}{2}(-2M_p) = 3.5\, M_p. \tag{35}$$

This upper limit for the value of WL agrees with that found by use of the virtual work method, Eq. 19, and the other limits as given by Eqs. 21 and 23 can be similarly derived from Eqs. 31 and 32.

To check the failure load for the mode of Fig. 11, Eqs. 30–33 are combined so as to eliminate the moments at sections 2, 5, 6 (after first putting $x = L$ in Eq. 31). This combination is easily made, the final result being

$$4WL = -M_1 + 2M_3 - M_4 + M_5 - M_8 + M_9 - M_{10}. \tag{36}$$

The maximum value of the right-hand side is $11M_p$, so that the upper limit $WL = 2.75\, M_p$ is obtained as before.

This check verifies only that the computation of the collapse load for a given mechanism is correct. In order to check that a mechanism actually is the correct solution of a problem it is necessary to prove that values of all moments can be found which do not violate any plasticity condition. In the present case the mechanism of Fig. 11 has seven hinges so that the determination of the complete set of moments follows directly from the equilibrium equations. The results are

$$M_2 = \tfrac{1}{2} M_p; \qquad M_6 = -M_p; \qquad M_7 = 1.75\, M_p. \tag{37}$$

Thus the fully plastic moment of no member is exceeded, and this verifies that the failure mode of Fig. 11 is the correct one, by the uniqueness theorem.

Checks of this sort by use of the equilibrium equations are always desirable. However, the procedure first suggested, of combining elementary mechanisms and computing collapse loads in each case by the virtual work method, seems the most advantageous practical scheme. When it is believed that all possible advantageous combinations of

mechanisms have been tried, then the result yielding the lowest failure load may be checked by using the equilibrium equations or bending moment diagrams.

It is necessary to make sure that all elementary mechanisms have been considered. This can be verified by using the fact that each elementary mechanism corresponds to an equation of equilibrium. Hence there are as many elementary mechanisms, together with rotations at joints where three or more members are connected, as there are independent equations of equilibrium relating the bending moments at cross-sections where the possibility of plastic hinges must be considered. Thus the number of elementary mechanisms is the total number of bending moments which must be considered in the collapse analysis, minus the number of redundants in the problem. For example in the illustrative problem solved, there are $10 - 6 = 4$ independent equations of equilibrium and hence elementary mechanisms (including joint rotations).

It will be evident that the method is not automatic, but requires some skill in visualizing advantageous combinations. However, this skill seems to be quite readily gained after a little experience. In practice it seems to be easy, especially for rectangular frames, to see what combinations lead to lowering of the collapse load, and also to see when all possible advantageous combinations have been tried. Clearly this method, like moment distribution in elastic frame analysis, can be learned only by practice. A few additional examples will be briefly indicated in Part II.

(To be continued.)

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Petroleum Coffee Bags.—Now the petroleum industry has come to the aid of the coffee drinker. The newest thing is individual coffee bags, packaged in saran film—a petroleum-derived plastic. The coffee bags are similar to tea bags, and are said to eliminate waste and to assure that each cup is fresh and brewed to individual taste.

Plastic-Coated Tubing Passes Corrosion Test.—Manufacturers of plastic coatings have been working with petroleum technicians on the problem of corrosion in oil well tubing. At completion of a five-month test, ordinary pipe was corroded deeply after being subjected to a daily flow of 100 barrels of oil and 5000 barrels of iodized salt water. Preliminary tests indicated that pipe coated with vinyl plastic was unaffected. Benefits from production research like this are ultimately passed on to consumers in the form of increasingly plentiful supplies of oil products.

Shock Testing of Packaged Goods.—The clinical history of a package subjected to shock and vibration tests can now be recorded immediately and permanently at the Packaging Development Branch of the Engineer Research and Development Laboratories, Fort Belvoir, Virginia.

An electronic device, designed and built to ERDL specifications by Reed Research, Inc., Washington, D. C., provides this record. Previous methods, while indicating the information required, did not make it at once available and permanent.

A record of this type, according to James A. Sargeant, ERDL, Chief, Packaging Development Laboratory, will be of great value in determining exact source and cause of damage occurring to a packaged article during transit and just how much and how long a package will protect its contents.

Another feature of the recorder, which embodies the best of many previous systems, is the 14-channel magnetic head. This permits recording of the activity in as many as 14 strategic parts of the package.

Accelerometers distributed throughout the test package indicate the frequency of vibration at various points. The signal is amplified, fed through the channel head and recorded on magnetic tape. Up to 14 channels may be recorded simultaneously at 60″ per second on one width of tape.

The tape is played back at 6″ per second into a magnetic compax recorder which by means of electric writing styli produces a permanent black line record on electrically conductive chart paper. By analyzing the wave forms traced by the styli, the shock, vibration, and accelerations inside the package at different stages of the test may be determined. Thus the effectiveness of cushioning and packaging may be accurately gauged.

The lower speed of the playback overcomes the mechanical difficulties encountered in attempting a direct transcription to paper of signals in the high frequency range.

The new device, designed to produce an accurate record of amplitudes and frequencies of shock and vibrations from 2 to 500 cycles per second and accelerations up to 80 g's, will be used in all shock and vibration tests at the Packaging Development Laboratory. Later it will be employed in tests of parachute and free fall packs released from airplanes. In this instance telemetering equipment will transmit the signal from the sensing devices to the recorder. The recorder will record up to 6 minutes and has a maximum playback of 1 hour.

ON THE FOUNDATIONS OF CERTAIN THEORIES OF TURBULENCE

BY

M. Z. v. KRZYWOBLOCKI[1]

SUMMARY

The author briefly discusses the foundations of modern theories of turbulence based on hydrodynamics equations. He shows that these theories seem to offer only a limited number of possible solutions.

1. FUNDAMENTAL EQUATIONS AND NOTIONS

Certain theories of turbulence assume the equations of hydrodynamics as the fundamental equations governing the turbulence pattern in the considered vector field of flow. There are four fundamental equations: motion, continuity, state, and energy. Let the symbol ρ denote the density, μ viscosity, k coefficient of heat conductivity, T temperature, etc.

Let us restrict ourselves to the correlation between two points P and P' and denote the quantities referring to the point P' by primes (u_i', T', μ', etc.). Introduce the temperature correlation function $(\overline{TT'})$, to be determined from the energy equation, the pressure correlation function $\overline{pp'}$ to be determined from the equation of state, etc. Not only the simple variables, but also the composite quantities, like Reynolds' number, etc., may constitute the correlation functions between two (or more) points. Let us first discuss the incompressible fluid flow, with ρ, μ, k being constants. In this case, the equations mentioned above simplify enormously. Particularly, the continuity equation reduces to $u_{i,i} = 0$ and equations of motion and continuity are fully independent of the two other equations.

Taylor-Kármán-Howarth consider only velocity correlation functions. This is constructionally achieved by the assumption that the turbulence is perfectly homogeneous and isotropic in velocity in the entire vector field of flow. The mathematical construction of the solution of the problem, in whatsoever form it is, may be obtained regardless of what assumptions are underlying the distribution of the temperature. The converse is not true. The temperature distribution depends upon the velocity through the equation of energy.

Kolmogoroff and others assume as the fundamental principle the transfer of energy. Theoretically at least, we must always take into account the total amount of the energy in the given energetic system, and not a part of it only. The law of conservation of energy refers to

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the total energy, whereas there may exist a transfer of energy from one form into another. *A priori*, at least theoretically, without justification one should not maintain that a part of energy is constant throughout the entire domain. Particularly, if one considers the dissipation of energy due to viscosity and neglects everything else, the omission of the heat conduction with $k \sim \mu c_p$ should be theoretically justified. The rate of change of the total kinetic energy ϵ_k is equal to:

$$\frac{D\epsilon_k}{Dt} = -\rho^{-1}pu_{i,i} + J\rho^{-1}(kT_{,i})_{,i} + \rho^{-1}\Phi + \tfrac{1}{2}\{(\textstyle\sum_i u_i^2)_{,t} + (\textstyle\sum_i u_i^2)_{,i}u_i\}. \tag{1}$$

Assuming, as usually in isotropic turbulence, that $\overline{u_i^2}$ is only a function of time, and neglecting the first and fourth terms on the right-hand side of (1) (quasi-steady flow, $u_{i,i} = 0$), we see that in the case of an incompressible fluid there are two terms on the right-hand side of (1). Kolmogoroff considers only one of them ($\rho^{-1}\Phi$), but since these theories of turbulence are applied to phenomena of magneto-hydrodynamic fields in the space, say, where the heat processes may be very important, let us discuss briefly the possibilities of attacking the problem with equation of energy taken into account:

2. POSSIBLE SOLUTIONS OF THE PROBLEM (INCOMPRESSIBLE FLUID)

The first possibility is to introduce generalized (composite) random variables. The equation for the propagation of the velocity correlation function remains the same as in the Kármán-Howarth approach. The equation of energy has the form:

$$Jc_v\theta(\overline{T^2})_{,t} = 2Jk\rho^{-1}\overline{T^2}(\theta_{,rr} + 2r\theta_{,r}) + \nu(\overline{\Phi T'} + \overline{\Phi' T}), \tag{2}$$

as one may easily verify with $\overline{TT'} = \overline{T^2(t)}\theta(r)$ and $\nu = \mu\rho^{-1}$. The last correlation function on the right-hand side is very complicated. It is not easy to relate it to velocity correlation functions and it presents great difficulties from the test standpoint. The second possibility is to introduce the independent random variables, for example, $\overline{\Phi(T + T')}$, and to simplify the problem enormously from the mathematical standpoint, but of course not from a physical standpoint. The third possibility is to neglect certain types of energy as Kolmogoroff did, and to assume the preserved part of the energy to be either a certain variable function or a constant (the last case was accepted by Kolmogoroff). The fourth possibility is to apply approximations of various types.

3. COMPRESSIBLE FLUID

The part of energy due to changes in volume may be considerable in certain cases and should not be neglected in some turbulence phenomena. In this case, the fundamental equations are very complicated,

particularly if one assumes $\mu = \mu(p, T)$, $k = k(p, T)$. The equation of continuity cannot serve directly in a simple way any more as a "relation-formula" between the scalar functions of one and the same tensor form as in Kármán-Howarth-Robertson theory. For example, let us try to find the value of the bilinear form $\overline{u_i u_j'}$. To this end multiply the continuity equation at P by u_j', that at P' by u_j, and subtract the products. Following the classical works of previous authors we have the relations: $\partial/\partial i' = \partial/\partial z_i$, $\partial/\partial i = -\partial/\partial z_i$, where $\vec{z} = \vec{r} = \overrightarrow{PP'}$. Thus, we obtain after applying the averaging process:

$$A_1 + A_2 + 2(\overline{u_i u'_j})_{,zi} = 0, \tag{3}$$

$$A_1 = \overline{\rho'_{,i}\rho'^{-1}u_j - \rho_{,i}\rho^{-1}u'_j}, \tag{3a}$$

$$A_2 = \overline{\rho'_{,i}\rho'^{-1}u'_i u_j - \rho_{,i}\rho^{-1}u_i u'_j}. \tag{3b}$$

The divergence of a simple bilinear correlation form $\overline{u_i u'_j}$ is defined in terms of more complicated functions A_1 and A_2. Moreover, these functions present great difficulties from the test standpoint. One can easily notice that the number of correlation functions $\overline{u_i u'_j \mu\mu'}$, $\overline{u_i u'_j TT'}$, $\overline{kk'TT'}$, etc., may be considerable. The second possibility is to introduce a certain type of "small perturbations" from a statistical standpoint, that is, to introduce the independent random variables. The statistical small perturbations are introduced into the domain of an incompressible flow domain, and represent the effect of compressibility. The velocity random variables may serve directly as the link between various terms of the equations of motion and energy. To the velocity random functions, $\overline{u_i u'_j}$, there are attached other random functions, such as $\overline{\mu\mu'}$, $\overline{\rho\rho'}$, $\overline{TT'}$, etc. By itself, this method of separating a mean product of random variables, $\overline{\rho\rho' u_i u'_j}$, say, into the product of mean random variables $\overline{\rho\rho'}\ \overline{u_i u'_j}$, involves the applicability of the solution to very small subdomains into which the entire flow domain should be divided. The third possibility is to apply some approximation solutions.

CONCLUSIONS

From the discussion presented above, it seems obvious that the turbulence theories, based on hydrodynamic equations, offer great possibilities in the case of an ideal perfectly homogeneous and isotropic turbulence in an incompressible fluid, but the rigorous applicability of these theories seems to decrease when a more actual picture from a physical standpoint is considered. Particularly in the case of a locally isotropic turbulence or a turbulence in a compressible fluid, or a turbulence where heat processes play a role, we are compelled to introduce far-reaching mathematical simplifications in order to obtain some solutions. Referring to composite random variables (Eq. 3), let us note

that, as is known, the mathematical expectation (mean value) of the product of random variables is expressible in terms of standard deviations and mathematical expectations of each of those random variables and their correlation coefficient. Although one can derive the corresponding equations, it is doubtful whether it is possible to solve them in practical applications unless some semi-experimental assumptions are introduced. High speed computing devices may be of a great help in particular cases, but general solutions cannot be obtained in such a way. Of course, a great number of particular cases solved may give certain hints towards general theory.

INTERFLECTIONS AND COLOR

BY

DOMINA EBERLE SPENCER [1] AND SHIRLEY E. SANBORN [2]

INTRODUCTION

The interflection method (1)[3] of calculating light distributions in rooms has proved to be of considerable practical importance. But, as applied in the past, color is neglected. How do multiple reflections change the apparent color of room surfaces? The purpose of this paper is to see in what way the predictions of the interflection theory should be modified if room surfaces are not achromatic but chromatic.

It has been proven mathematically by Hodara (2) that the quantity of light obtained after a number of interflections is always greater with chromatic surfaces than with achromatic surfaces of the same total luminous reflectance. Experience in choosing colors for large surfaces in rooms indicates also that the effective color of the walls tends to increase in chroma, compared with the small sample used in planning the design. Thus both effective reflectance and color are affected by interflections.

The present paper extends the interflection theory to the calculation of trichromatic coordinates for light from wall, ceiling, and floor, thereby providing a physical basis for assessing the effect of chromatic room surfaces. Quantitative examples are given in typical cases.

INTERFLECTIONS IN THE INFINITE ROOM

As a simple example, consider an infinite room lighted by a luminous ceiling. All surfaces are assumed to be perfectly diffusing. The initial radiation from the ceiling is specified by a phengosage curve, $J_0(\lambda)$ (watt m^{-2} μ^{-1}), while the spectral reflectances of ceiling[4] and floor are designated by the functions $\rho_2(\lambda)$ and $\rho_3(\lambda)$. The final values of phengosage[5] from ceiling and floor are

$$J_2(\lambda) = \frac{J_0(\lambda)}{1 - \rho_2(\lambda)\rho_3(\lambda)}, \tag{1}$$

$$J_3(\lambda) = \frac{\rho_3(\lambda)J_0(\lambda)}{1 - \rho_2(\lambda)\rho_3(\lambda)}. \tag{2}$$

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

[4] As in previous papers on interflections, subscripts 1, 2, and 3 refer to wall, ceiling, and floor, respectively.

[5] See Section 5.03 of *Lighting Design* (ref. 1).

The trichromatic specification of the light from the ceiling is given by the three coordinates:

$$X_2 = \int_0^\infty \frac{\bar{x}(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)},$$

$$Y_2 = \int_0^\infty \frac{\bar{y}(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)}, \tag{3}$$

$$Z_2 = \int_0^\infty \frac{\bar{z}(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)},$$

while the light from the floor is specified by

$$X_3 = \int_0^\infty \frac{\bar{x}(\lambda)\rho_3(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)},$$

$$Y_3 = \int_0^\infty \frac{\bar{y}(\lambda)\rho_3(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)}, \tag{4}$$

$$Z_3 = \int_0^\infty \frac{\bar{z}(\lambda)\rho_3(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)}.$$

Without interflections the helios of the ceiling would be

$$H_{02} = c\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda, \tag{5}$$

where H_{02} is expressed in blondels, and c is the conversion factor 647.8. The actual values of helios H_2 and H_3 of floor and ceiling are expressed in terms of H_{02} by the relations,

$$\frac{H_2}{H_{02}} = \frac{\int_0^\infty \frac{\bar{y}(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)}}{\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda}, \tag{6}$$

$$\frac{H_3}{H_{02}} = \frac{\int_0^\infty \frac{\bar{y}(\lambda)\rho_3(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)}}{\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda}. \tag{7}$$

The interflectance f of the room [pharos (lumen) to principal surface/pharos (lumen) from luminous ceiling] becomes

$$f = \frac{\int_0^\infty \frac{\bar{y}(\lambda)J_0(\lambda)d\lambda}{1-\rho_2(\lambda)\rho_3(\lambda)}}{\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda}. \tag{8}$$

The above expressions, which include the effect of interflections among chromatic surfaces, with an arbitrary illuminant, are now com-

pared with the corresponding equations which neglect color. It is customary to specify the reflectances of ceiling and floor by the constants

$$\rho_2 = \frac{\int_0^\infty \bar{y}(\lambda)\rho_2(\lambda)J_0(\lambda)d\lambda}{\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda} \tag{9}$$

and

$$\rho_3 = \frac{\int_0^\infty \bar{y}(\lambda)\rho_3(\lambda)J_0(\lambda)d\lambda}{\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda}. \tag{10}$$

In the usual treatment, these total reflectances are considered as constants, independent of wavelength. The resulting helios ratios and interflectance are

$$\frac{H_2}{H_{02}} = \frac{1}{1 - \rho_2\rho_3}, \tag{6a}$$

$$\frac{H_3}{H_{02}} = \frac{\rho_3}{1 - \rho_2\rho_3}, \tag{7a}$$

$$f = \frac{1}{1 - \rho_2\rho_3}. \tag{8a}$$

Clearly, the customary treatment makes a rather gross oversimplification which might introduce significant errors.

INTERFLECTIONS IN A FINITE ROOM

The procedure for the finite room is similar to that for the infinite room, the only difference being that the expressions become much more cumbersome. Attention will be confined to a room lighted by a luminous ceiling. The room surfaces are assumed to be perfectly diffusing, with wall, ceiling, and floor reflectances ρ_1, ρ_2, and ρ_3. In previous papers (1), equations have been derived for H_{1M}/H_{02}, H_2/H_{02}, H_3/H_{02}, and f as a function of ρ_1, ρ_2, ρ_3 and k_r. Here

H_{1M} = average helios of wall midway between ceiling and floor,
H_2 = average helios of ceiling,
H_3 = average helios of floor,
f = interflectance of room, and
H_{02} = helios of ceiling without interflections.

The quantity k_r (called domance) specifies the shape of the room:

$$k_r = \frac{h(l + w)}{2lw} = \frac{\text{wall area}}{4 \text{ floor area}}, \tag{11}$$

where h, l, and w are height, length, and width of the room, respectively.

If color is considered, the above expressions are valid at each wavelength. Thus the previous equations (1) are still true, but each quantity becomes a function also of wavelength:

$$H_{1M}/H_{02}(\lambda, k_r)$$
$$H_2/H_{02}(\lambda, k_r)$$
$$H_3/H_{02}(\lambda, k_r)$$
$$f(\lambda, k_r).$$

The phengosage from wall, ceiling, or floor is

$$J_{1M}(\lambda) = \frac{H_{1M}}{H_{02}}(\lambda, k_r)J_0(\lambda), \tag{12}$$

$$J_2(\lambda) = \frac{H_2}{H_{02}}(\lambda, k_r)J_0(\lambda), \tag{13}$$

$$J_3(\lambda) = \frac{H_3}{H_{02}}(\lambda, k_r)J_0(\lambda), \tag{14}$$

while the phengosage for the light incident on the principal surface is

$$J(\lambda) = f(\lambda, k_r)J_0(\lambda). \tag{15}$$

The trichromatic coordinates for light from wall, ceiling, and floor are calculated from the equations,

$$X_{1M} = \int_0^\infty \bar{x}(\lambda)\frac{H_{1M}}{H_{02}}(\lambda, k_r)J_0(\lambda)d\lambda,$$
$$Y_{1M} = \int_0^\infty \bar{y}(\lambda)\frac{H_{1M}}{H_{02}}(\lambda, k_r)J_0(\lambda)d\lambda, \tag{16}$$
$$Z_{1M} = \int_0^\infty \bar{z}(\lambda)\frac{H_{1M}}{H_{02}}(\lambda, k_r)J_0(\lambda)d\lambda,$$

$$X_2 = \int_0^\infty \bar{x}(\lambda)\frac{H_2}{H_{02}}(\lambda, k_r)J_0(\lambda)d\lambda,$$
$$Y_2 = \int_0^\infty \bar{y}(\lambda)\frac{H_2}{H_{02}}(\lambda, k_r)J_0(\lambda)d\lambda, \tag{17}$$
$$Z_2 = \int_0^\infty \bar{z}(\lambda)\frac{H_2}{H_{02}}(\lambda, k_r)J_0(\lambda)d\lambda,$$

$$X_3 = \int_0^\infty \bar{x}(\lambda) \frac{H_3}{H_{02}} (\lambda, k_r) J_0(\lambda) d\lambda,$$

$$Y_3 = \int_0^\infty \bar{y}(\lambda) \frac{H_3}{H_{02}} (\lambda, k_r) J_0(\lambda) d\lambda, \tag{18}$$

$$Z_3 = \int_0^\infty \bar{z}(\lambda) \frac{H_3}{H_{02}} (\lambda, k_r) J_0(\lambda) d\lambda.$$

Similarly, the incident radiation on the principal surface is specified by

$$X = \int_0^\infty \bar{x}(\lambda) f(\lambda, k_r) J_0(\lambda) d\lambda,$$

$$Y = \int_0^\infty \bar{y}(\lambda) f(\lambda, k_r) J_0(\lambda) d\lambda, \tag{19}$$

$$Z = \int_0^\infty \bar{z}(\lambda) f(\lambda, k_r) J_0(\lambda) d\lambda.$$

Therefore,

$$\frac{H_{1M}}{H_{02}} = \frac{\int_0^\infty \bar{y}(\lambda) \frac{H_{1M}}{H_{02}} (\lambda, k_r) J_0(\lambda) d\lambda}{\int_0^\infty \bar{y}(\lambda) J_0(\lambda) d\lambda}, \tag{20}$$

$$\frac{H_2}{H_{02}} = \frac{\int_0^\infty \bar{y}(\lambda) \frac{H_2}{H_{02}} (\lambda, k_r) J_0(\lambda) d\lambda}{\int_0^\infty \bar{y}(\lambda) J_0(\lambda) d\lambda}, \tag{21}$$

$$\frac{H_3}{H_{02}} = \frac{\int_0^\infty \bar{y}(\lambda) \frac{H_3}{H_{02}} (\lambda, k_r) J_0(\lambda) d\lambda}{\int_0^\infty \bar{y}(\lambda) J_0(\lambda) d\lambda}. \tag{22}$$

The interflectance of the room is likewise given by

$$f = \frac{\int_0^\infty \bar{y}(\lambda) f(\lambda, k_r) J_0(\lambda) d\lambda}{\int_0^\infty \bar{y}(\lambda) J_0(\lambda) d\lambda}. \tag{23}$$

Comparing with ordinary interflection theory, we find

$$\rho_1 = \frac{\int_0^\infty \bar{y}(\lambda)\rho_1(\lambda)J_0(\lambda)d\lambda}{\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda},$$

$$\rho_2 = \frac{\int_0^\infty \bar{y}(\lambda)\rho_2(\lambda)J_0(\lambda)d\lambda}{\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda}, \tag{24}$$

$$\rho_3 = \frac{\int_0^\infty \bar{y}(\lambda)\rho_3(\lambda)J_0(\lambda)d\lambda}{\int_0^\infty \bar{y}(\lambda)J_0(\lambda)d\lambda},$$

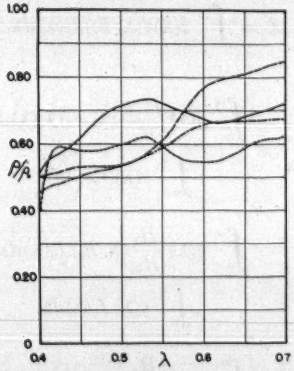

FIG. 1. Spectral reflectances of a set of paints.

coral
tan
green
blue

$$\frac{H_{1M}}{H_{02}} = \frac{H_{1M}}{H_{02}}(\rho_1, \rho_2, \rho_3, k_r), \tag{20a}$$

$$\frac{H_2}{H_{02}} = \frac{H_2}{H_{02}}(\rho_1, \rho_2, \rho_3, k_r), \tag{21a}$$

$$\frac{H_3}{H_{02}} = \frac{H_3}{H_{02}}(\rho_1, \rho_2, \rho_3, k_r), \tag{22a}$$

$$f = f(\rho_1, \rho_2, \rho_3, k_r). \tag{23a}$$

The detailed equations are given in several publications (1) and need not be repeated here.

EXAMPLES

To determine the order of magnitude of quantitative effects to be expected from the inclusion of color in the study of interflections, we have computed two typical examples. Colors were chosen from actual

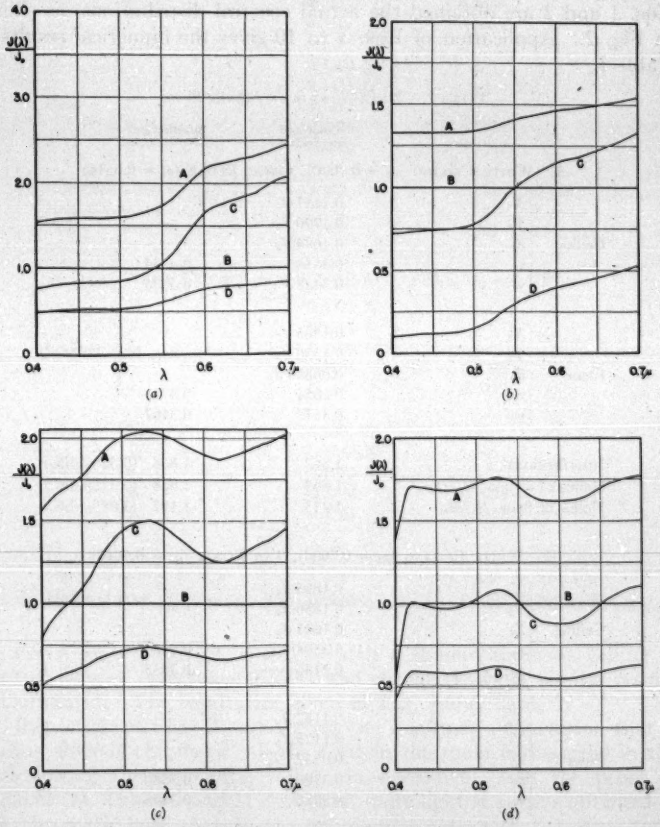

FIG. 2. Interflections in an infinite room with luminous ceiling. Phengosage (spectral distribution curves) for *A* light from ceiling with interflections, *B* light from ceiling without interflections, *C* light from floor with interflections, *D* light from floor without interflections.

(*a*) coral floor
(*b*) tan floor
(*c*) green floor
(*d*) blue floor

industrial paints (3). Spectral reflectances of the four paints chosen for our study are shown in Fig. 1. Since ceiling lighting appears to be best for most applications, this form of lighting is employed in all examples.

Consider first an infinite room. The luminous ceiling is assumed to have a constant reflectance of 0.7000, and the light source has an equi-energy spectrum. The colors of Fig. 1 are utilized for the floor. From Eqs. 1 and 2 are obtained the actual spectral distributions, as shown in Fig. 2. Application of Eqs. 3 to 10 gives the numerical results of Table I.

TABLE I.—*Interflections in an Infinite Room.*

		Color and interflections	Interflections (Color neglected)
A. WHITE CEILING ($\rho_2 = 0.7000$), CORAL FLOOR ($\rho_1 = 0.6514$)			
Ceiling	X_2	0.2121 J_0	—
	Y_2	0.1990 J_0	—
	Z_2	0.1694 J_0	—
	x_2	0.3653	0.3334
	y_2	0.3428	0.3335
Floor	X_1	0.1505 J_0	—
	Y_1	0.1320 J_0	—
	Z_1	0.08880 J_0	—
	x_1	0.4054	0.3711
	y_1	0.3555	0.3467
Interflectance, f		1.862	1.838 (0.32% diff.)
Helios of ceiling, H_2/H_{02}		1.862	1.838 (0.32% diff.)
Helios of floor, H_1/H_{01}		1.235	1.197 (3.08% diff.)
B. WHITE CEILING ($\rho_2 = 0.7000$), TAN FLOOR ($\rho_3 = 0.6040$)			
Ceiling	X_2	0.1895 J_0	—
	Y_2	0.1856 J_0	—
	Z_2	0.1651 J_0	—
	x_2	0.3508	0.3334
	y_2	0.3436	0.3335
Floor	X_3	0.1182 J_0	—
	Y_3	0.1125 J_0	—
	Z_3	0.08323 J_0	—
	x_3	0.3765	0.3586
	y_3	0.3584	0.3495
Interflectance, f		1.737	1.732 (0.29% diff.)
Helios of ceiling, H_2/H_{02}		1.737	1.732 (0.29% diff.)
Helios of floor, H_3/H_{03}		1.053	1.046 (0.66% diff.)

TABLE I.—*Continued.*

		Color and interflections	Interflections (Color neglected)
C. WHITE CEILING ($\rho_2 = 0.7000$), GREEN FLOOR ($\rho_3 = 0.7043$)			
Ceiling	X_2	0.2034 J_0	—
	Y_2	0.2108 J_0	—
	Z_2	0.1920 J_0	—
	x_2	0.3355	0.3334
	y_2	0.3477	0.3336
Floor	X_3	0.1380 J_0	—
	Y_3	0.1487 J_0	—
	Z_3	0.1218 J_0	—
	x_3	0.3378	0.3376
	y_3	0.3640	0.3502
Interflectance, f		1.972	1.972 (0% diff.)
Helios of ceiling, H_2/H_{02}		1.972	1.972 (0% diff.)
Helios of floor, H_3/H_{03}		1.392	1.389 (0.22% diff.)
D. WHITE CEILING ($\rho_2 = 0.7000$), BLUE FLOOR ($\rho_3 = 0.5785$)			
Ceiling	X_2	0.1773 J_0	—
	Y_2	0.1801 J_0	—
	Z_2	0.1806 J_0	—
	x_2	0.3295	0.3334
	y_2	0.3347	0.3335
Floor	X_3	0.1001 J_0	—
	Y_3	0.1042 J_0	—
	Z_3	0.1052 J_0	—
	x_3	0.3234	0.3275
	y_3	0.3367	0.3353
Interflectance, f		1.685	1.680 (0.30% diff.)
Helios of ceiling, H_2/H_{02}		1.685	1.680 (0.30% diff.)
Helios of floor, H_3/H_{03}		0.9746	0.9721 (0.26% diff.)

For a finite room ($k_r = 0.700$) with a similar luminous ceiling, a neutral floor of reflectance 0.3000, and chromatic walls, similar results are obtained. The results are given in Fig. 3 and Table II.

Inspection of Figs. 2 and 3 gives the qualitative conclusion that if wall or floor is chromatic, all the light in the room is distorted in the direction of the chromatic color. In agreement with the proof of Hodara (2), Tables I and II show that more light is always obtained in the chromatic case than in the achromatic case with the same total reflectances. The difference, however, is small. It is never greater than 3 per cent and is generally less than 1 per cent in the examples considered in this paper.

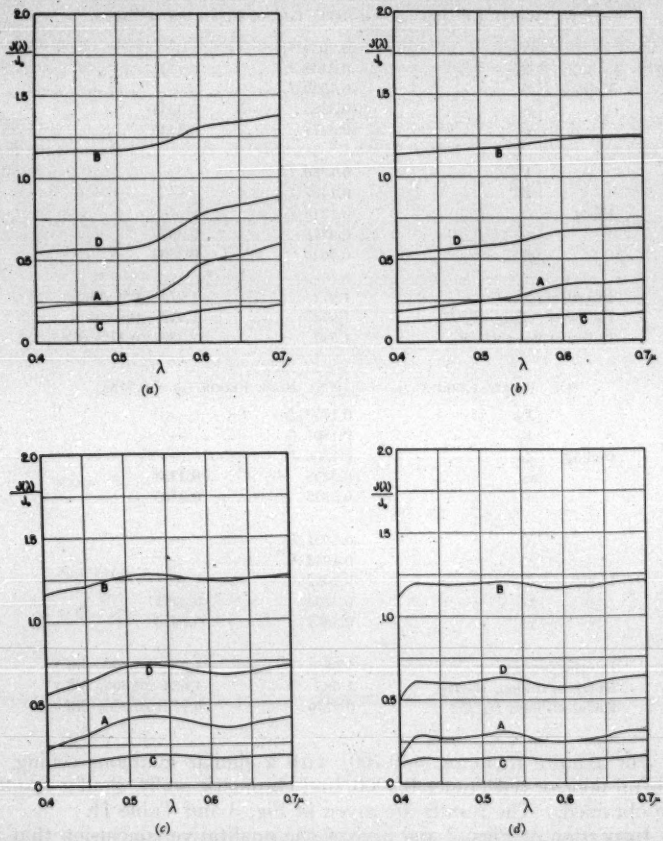

FIG. 3. Interflections in a finite room with luminous ceiling. Phengosage (spectral distribution curves) for light from *A* wall, *B* ceiling, *C* floor with interflections, and *D* incident pharosage.

(*a*) coral walls
(*b*) tan walls
(*c*) green walls
(*d*) blue walls

TABLE II.—*Interflections in a Finite Room.*

		Color and interflections	Interflections (Color neglected)
A. CORAL WALLS ($\rho_1 = 0.6514$), WHITE CEILING ($\rho_2 = 0.7000$), GRAY FLOOR ($\rho_3 = 0.3000$)			
Wall	X_1	$0.04279\ J_0$	—
	Y_1	$0.03747\ J_0$	—
	Z_1	$0.02527\ J_0$	—
	x_1	0.4055	0.3711
	y_1	0.3551	0.3467
Ceiling	X_2	$0.1351\ J_0$	—
	Y_2	$0.1319\ J_0$	—
	Z_2	$0.1246\ J_0$	—
	x_2	0.3449	0.3335
	y_2	0.3370	0.3336
Floor	X_3	$0.01907\ J_0$	—
	Y_3	$0.01772\ J_0$	—
	Z_3	$0.01433\ J_0$	—
	x_3	0.3731	0.3335
	y_3	0.3466	0.3336
Incident pharosage	X	$0.07646\ J_0$	—
	Y	$0.07155\ J_0$	—
	Z	$0.06059\ J_0$	—
	x	0.3666	0.3335
	y	0.3430	0.3336
Interflectance, f		0.6695	0.662 (1.12% diff.)
Wall helios, H_{1M}/H_{02}		0.3506	0.339 (3.32% diff.)
Ceiling helios, H_2/H_{02}		1.235	1.227 (0.65% diff.)
Floor helios, H_3/H_{02}		0.1658	0.1608 (3.02% diff.)
B. TAN WALLS ($\rho_1 = 0.6040$), WHITE CEILING ($\rho_2 = 0.7000$), GRAY FLOOR ($\rho_3 = 0.3000$)			
Wall	X_1	$0.03332\ J_0$	—
	Y_1	$0.03178\ J_0$	—
	Z_1	$0.02334\ J_0$	—
	x_1	0.3768	0.3586
	y_1	0.3594	0.3495
Ceiling	X_2	$0.1294\ J_0$	—
	Y_2	$0.1286\ J_0$	—
	Z_2	$0.1236\ J_0$	—
	x_2	0.3392	0.3334
	y_2	0.3370	0.3335
Floor	X_3	$0.01643\ J_0$	—
	Y_3	$0.01617\ J_0$	—
	Z_3	$0.01407\ J_0$	—
	x_3	0.3521	0.3335
	y_3	0.3464	0.3336

TABLE II.—*Continued.*

		Color and interflections	Interflections (Color neglected)
Incident pharosage	X	0.06815 J_0	—
	Y	0.06679 J_0	—
	Z	0.05928 J_0	—
	x	0.3509	0.3335
	y	0.3439	0.3336
Interflectance, f		0.6250	0.621 (0.64% diff.)
Wall helios, H_{1M}/H_{02}		0.2974	0.297 (0.13% diff.)
Ceiling helios, H_2/H_{02}		1.203	1.203 (0% diff.)
Floor helios, H_3/H_{02}		0.1513	0.151 (0.20% diff.)

C. GREEN WALLS ($\rho_1 = 0.7043$), WHITE CEILING ($\rho_2 = 0.7000$), GRAY FLOOR ($\rho_3 = 0.3000$)

Wall	X_1	0.03900 J_0	—
	Y_1	0.04197 J_0	—
	Z_1	0.03469 J_0	—
	x_1	0.3372	0.3376
	y_1	0.3629	0.3502
Ceiling	X_2	0.1328 J_0	—
	Y_2	0.1347 J_0	—
	Z_2	0.1301 J_0	—
	x_2	0.3339	0.3334
	y_2	0.3388	0.3335
Floor	X_3	0.01798 J_0	—
	Y_3	0.01890 J_0	—
	Z_3	0.01661 J_0	—
	x_3	0.3361	0.3334
	y_3	0.3533	0.3335
Incident pharosage	X	0.07286 J_0	—
	Y	0.07562 J_0	—
	Z	0.06877 J_0	—
	x	0.3354	0.3334
	y	0.3481	0.3335
Interflectance, f		0.7076	0.707 (0.09% diff.)
Wall helios, H_{1M}/H_{02}		0.3928	0.392 (0.20% diff.)
Ceiling helios, H_2/H_{02}		1.261	1.259 (0.16% diff.)
Floor helios, H_3/H_{02}		0.1769	0.176 (0.51% diff.)

D. BLUE WALLS ($\rho_1 = 0.5785$), WHITE CEILING ($\rho_2 = 0.7000$), GRAY FLOOR ($\rho_3 = 0.3000$)

Wall	X_1	0.02821 J_0	—
	Y_1	0.02931 J_0	—
	Z_1	0.02958 J_0	—
	x_1	0.3239	0.3275
	y_1	0.3365	0.3353

TABLE II.—*Continued.*

		Color and interflections	Interflections (Color neglected)
Ceiling	X_2	0.1265 J_0	—
	Y_2	0.1272 J_0	—
	Z_2	0.1276 J_0	—
	x_2	0.3318	0.3334
	y_2	0.3336	0.3335
Floor	X_3	0.01526 J_0	—
	Y_3	0.01578 J_0	—
	Z_3	0.01565 J_0	—
	x_3	0.3269	0.3334
	y_3	0.3379	0.3335
Incident pharosage	X	0.06351 J_0	—
	Y	0.06467 J_0	—
	Z	0.06451 J_0	—
	x	0.3296	0.3334
	y	0.3356	0.3335
Interflectance, f		0.6052	0.6000 (0.83% diff.)
Wall helios, H_{1M}/H_{02}		0.2743	0.273 (0.48% diff.)
Ceiling helios, H_2/H_{02}		1.190	1.188 (0.17% diff.)
Floor helios, H_3/H_{02}		0.1476	0.144 (2.44% diff.)

Figure 4 indicates the change in the color resulting from interflections. The general conclusion is that all surfaces take on a higher chroma and that the shift is of the order of two Munsell chroma steps.

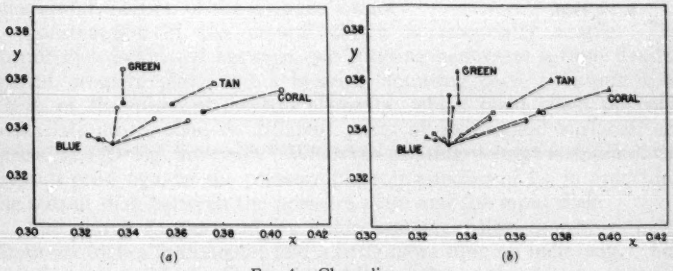

FIG. 4. Chros diagram.

x and y for light from
(*a*) Infinite room with chromatic floor
● ceiling, neglecting interflections
○ ceiling, with interflections
■ floor, neglecting interflections
□ floor, with interflections
(*b*) Finite room with chromatic walls
▲ walls, neglecting interflections
△ walls, with interflections
● ceiling, neglecting interflections
○ ceiling, with interflections
● floor, neglecting interflections
□ floor, with interflections
● incident pharosage, neglecting interflections
D incident pharosage, with interflections

CONCLUSIONS

The interflection method of lighting calculations has previously neglected color. In this paper, the equations are developed by which both the magnitude and the color of the distribution of light in rooms can be predetermined. The evaluation of typical numerical examples results in the following conclusions:

(*a*) The quantity of light calculated by the ordinary interflection theory (neglecting color) is always slightly low, but by only a few percent. Thus the ordinary calculations are conservative and the errors are smaller than the experimental errors in measurement.

(*b*) The color of all surfaces is shifted to higher chroma by about two Munsell chroma steps. The practical implications of this result are that caution should be used in choosing the colors of large room surfaces. The interflections of light will make the chroma of the wall paint or floor covering appreciably higher than that of a small sample viewed under the same illuminant.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

A HIGH-SPEED CRYSTAL CLUTCH

An experimental crystal clutch recently developed at the National Bureau of Standards is believed to be the first of its type. In the new clutch, application of a direct-current voltage to the electrodes of three "Bimorph"[1] piezoelectric crystal elements causes bending of the elements; this bending presses the clutch output disk against the rotating input disk. The new experimental unit was devised by Ernest Codier of the NBS staff as part of a program for the development of fast-acting clutches suitable for use in high-speed computers.

Distinguishing features of the crystal clutch are high speed of response and almost negligible current drain. No current flows, other than insulation leakage, after the applied voltage has charged the capacity of the crystals. The clutch therefore creates no magnetic field, an advantage in some applications.

In the only clutch model thus far constructed, the output shaft delivered useful torque in as little as 0.2 millisecond after voltage was applied. Output torque of the engaged clutch was approximately 16.5 ounce-inches at 400 volts excitation, or 21 ounce-inches for 500 volts. The no-voltage drag torque, however, was about 7.5 ounce-inches, a substantial fraction of the engaged torque.

Construction of the crystal clutch is essentially simple. The output disk is located between two rotating members: a thin, flexible crystal pressure plate and a heavier mounting plate or input disk. Three of the Bimorph crystal elements, which bend when opposite potentials are applied to different parts of the crystal surfaces, are spaced at 120-deg. intervals. When the exciting voltage is applied, the crystals press against the pressure plate at a radius of 1.5 in., pinching the output disk between the pressure plate and the input disk.

The clutch proper and its immediate mounting occupy a space about six inches in diameter and a little more than an inch long. The complete assembly includes, in addition, internal bearings for the output shaft, bearings and mounts for the entire assembly, a drive pulley, and slip rings for transmitting the exciting voltage to the rotating assembly.

The new clutch was tested for speed of response by placing the needle of an ordinary crystal phonograph pickup in a small dimple in the clutch output shaft. By connecting the pickup output to the

* Communicated by the Director.

[1] Product of Brush Development Company.

vertical input of a cathode-ray oscilloscope, and connecting the clutch excitation voltage to the horizontal input, the time lag between the application of the exciting voltage and the resulting motion of the output shaft was observed. Alternatively, by connecting the Bimorph terminals to the vertical plates of the oscilloscope, it was possible to observe the voltage rise as the capacitance of the Bimorph crystals became charged.

Several factors enter into the design of a clutch of this type. Speed of response, the principal design goal, is related to the inertia and loading of the output system, the available torque, the distance the crystal pressure point must move before it begins to pinch the output disk, and the resonant frequency of the crystal itself. It is the resonant frequency that sets the ultimate limit for speed of response.

The dimensions of the Bimorph crystal determine not only its resonant frequency but also its sensitivity. The resonant frequency, in kilocycles, of a Rochelle-salt crystal Bimorph is given by $26\ T/LW$, and the sensitivity in mils per kilovolt is approximately $0.08\ LW/T^2$, where L, W, and T are crystal length, width, and thickness, in inches. Since increased crystal thickness is favorable to high resonant frequency but unfavorable to high sensitivity, design dimensions must be matters of compromise and judgment.

If resistance is present in the excitation circuit, the voltage appearing across the crystals will rise exponentially while the crystal capacity (about 0.0065 μf) is charged. If the voltage rise is excessively slow, speed of response will of course be impaired. It might be thought that if resistance were eliminated, so as to give very fast voltage rise, speed of response would be a maximum. This does not prove to be the case, however. Instead, an irregular output motion appears, probably due to bouncing of the crystal against the output disk, while response time shows no significant improvement.

Rochelle-salt crystal elements are used in the present clutch because they are readily available and highly sensitive. Rochelle salt has several disadvantages, however: it is easily fractured, melts at 55° C., and will deliquesce unless protected from humidity. Barium titanate crystals, a logical alternative, should provide a mechanically rugged clutch usable at temperatures up to 100° C. Barium titanate is less sensitive than Rochelle salt, however, and also has a higher capacity.

Obviously, the crystal clutch developed at NBS does not have wide applicability. Yet its high speed and negligible current drain could be very valuable in a few highly specialized but nevertheless important instrumentation functions. In special photographic recording apparatus, for instance, the crystal clutch could be used to move motion picture film in fast response to an actuating voltage. The experimental clutch model could undoubtedly be improved upon; in fact, several desirable modifications have already become apparent.

AIR-GRAVEL CONCRETE

A lightweight concrete recently developed by R. C. Valore, Jr. and W. C. Green of the National Bureau of Standards substitutes air bubbles for sand. Suitable for the construction of monolithic walls and other elements of small structures, this "air-gravel concrete" grew out of a request from the U. S. Bureau of Plant Industry, Soils, and Agricultural Engineering for a material particularly adaptable to farm buildings.[1] The many tiny air bubbles enclosed within the material not only replace the sand used in ordinary dense concrete, but also make the product lighter in weight, a better insulator against heat or cold, and more resistant to water penetration.

Concrete normally contains a coarse aggregate such as gravel or crushed stone held together by a binder of mortar consisting of sand and cement-water paste. In the air-gravel concrete the fine aggregate or sand is replaced by air bubbles, which act as a "lubricant" and produce a very workable mixture. High-air-content concrete is prepared in conventional mixing equipment in the normal way, except that an industrial aerating agent or a detergent is added to the mix to cause it to foam by entraining finely-divided air cells in the cement-water paste. Entrained air may constitute up to 45 per cent of the volume of the concrete prepared at the Bureau, and "pea gravel" forms the dense aggregate.

One of the important functions of fine aggregate sand in conventional concrete is to produce workability, which facilitates placing in forms and assures homogeneity in structural members. Results of an investigation at NBS indicate that the replacement of conventional fine aggregate by entrained air likewise produces workable mixtures having relatively low water-cement ratios. The mixtures studied consisted of a high-early-strength portland cement, siliceous pea gravel (No. 4 to $\frac{3}{8}$-in. sieve fraction), water, and an air-entraining agent. Two different air-entraining agents (foam stabilizers) were used: agent V, a proprietary neutralized resin; and agent X, a proprietary sodium lauryl sulfate-type detergent. The falling action of the gravel particles during mixing in an ordinary rotating tilt-drum mixer produced the entrainment of air in the cement-water paste, and the air-entraining agent rendered the foamed paste stable.

Air contents as high as 45 per cent were found possible in concretes containing no sand, while there appeared to a "ceiling" of about 30 per cent in the air content of experimental mixtures containing both sand and gravel. However, the amount of entrained air, upon which the density depends, was limited critically by the minimum compressive strength requirement of 500 lb./in^2. The lowest cement content

[1] For further technical details see "Air Replaces Sand in 'No-Fines' Concrete," by Rudolph C. Valore, Jr. and William C. Green, *Journal of American Concrete Institute*, June 1951; *Proc.*, Vol. 47, p. 833.

permitting the attainment of the required strength in an "air-gravel" concrete was about 3¼ bags of cement per cu. yd. of concrete. In this "lean" mix (mix A) it was necessary to maintain an air content of about 25 per cent to provide the necessary workability and strength. A second mix (mix C), containing about 5.6 bags of cement per cu. yd. of concrete, provided the required strength for air contents as high as 29 per cent.

The compressive strength was sensitive to variations in air content; an increase of 1 per cent in air content was generally accompanied by a decrease of about 100 lb./in.² in compressive strength for both mix A and mix C. The indicated density of concretes having a compressive strength at 28 days of 500 lb./in.² was about 113 lb./cu. ft. for mix A, and 105 and 109 lb./cu. ft. respectively for mix C concretes made with the resin and detergent air-entraining agents, as compared with 140 to 145 lb./cu. ft. for conventional (nonaerated) sand-gravel concretes. While the compressive strengths were only one-fourth to one-fifth the values obtained for nonaerated concretes, the moisture properties and thermal conductivity of the air-gravel concretes compared favorably with the values obtained for the denser concretes.

The mean water absorption of the lightweight concretes upon soaking for 24 hours at room temperature was about 9 per cent (by volume) for all mixes, compared with 11 to 15 per cent for conventional concretes of similar materials and cement contents. Saturation coefficients, calculated as the ratio of the 24-hr. room temperature absorption, to the 5-hr. boiling absorption, ranged from 0.22 to 0.36, or about one-third the magnitude of values previously reported for nonaerated sand-gravel concretes. The saturation coefficient purports to represent the ratio of the pore volume easily filled by water to total pore volume. The drying shrinkage of the air-gravel concretes did not differ significantly from values obtained for nonaerated sand-gravel concretes, ranging from 0.04 to 0.07 per cent for 2- by 12-in. prisms after 180 days drying.

Of 13 specimens of mix A and mix C concretes subjected to 100 cycles of laboratory freezing (0° F. air) and thawing (70° F. water), only one specimen (mix C) failed at 40 cycles. Two specimens of nonaerated sand-gravel concrete failed at 60-cycles.

The thermal conductivity of these concretes, as determined in the NBS guarded hot plate test, ranged from about 5 to 6 Btu [hr. ft.² (°F/in.)]; a value obtained in tests of a nonaerated sand-gravel concrete was 9. A value often cited for conventional concretes is 12, but values ranging from 7 to 16 have been reported by various observers.

The air-gravel concretes developed at the National Bureau of Standards are considered experimental at the present time. The advantages in reduced density and thermal conductivity are appreciable but are gained only at a great sacrifice in strength. Greater advantages might be gained with higher cement contents and higher air contents, but the relatively high density of conventional aggregates imposes

certain limits. The use of lightweight aggregates blended with dense aggregates in relatively high-air-content concretes may be a means of obtaining worth-while reductions in density and thermal conductivity while maintaining low-order drying shrinkages.

SILICONE RESIN PIGMENT FOR CLINICAL THERMOMETERS

As the result of research at the National Bureau of Standards, a durable, lasting pigment is now available for marking clinical thermometers. Developed by R. J. Welch and F. W. Reinhart of the NBS staff, the new marking compound is a silicone resin formulation that withstands cleansing with soap and phenol solutions far better than any other known pigment.

Thermometers used in hospitals must undergo constant immersion in detergents or soap solutions and germicides such as phenol. Loss of pigment due to the action of these cleansing agents has been a major cause of thermometer failure. The Hospital Bureau of Standards and Supplies and the American Hospital Association therefore requested the National Bureau of Standards to make a study of the situation with a view to the development of a pigment that would withstand normal hospital usage. A large number of formulations were investigated, as well as various methods of etching. Finally the new silicone formula was developed, and long-term immersion tests showed that it was the answer to the problem.

The basic ingredient of the NBS marking compound is a silicone resin.[1] To 15 parts by weight of the resin are added five parts of bone black, one part of normal butyl alcohol, and one part of toluene. The mixture, which in the trade is designated "pigment," is ground in a ball or pebble mill; this disperses the bone black in the resin and results in a soft, buttery consistency. If a longer drying time is desired, xylene may be substituted either in whole or in part for the toluene. The bone black in the compound settles partially on standing for one to three days. However, since the black does not cake, it is readily redistributed by stirring.

A small round brush with short bristles is used to apply the pigment to the thermometer stem. The thermometers are then allowed to dry in the open air at a temperature above 70° F. After four days the thermometers may be immersed in a water bath to test their accuracy, and at the end of two weeks they may be exposed to disinfectants.

In simulated service tests at the National Bureau of Standards, thermometers marked with the new NBS formulation successfully withstood more than seven months' continuous or intermittent exposure to a large number of disinfectants, soap solutions, detergents, and other chemicals.

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[1] Designated GE 9980.

THE FRANKLIN INSTITUTE

MEDAL DAY MEETING

Wednesday, October 17, 1951

The Annual Medal Day Exercises of The Franklin Institute were held on Wednesday, October 17, 1951, in the Benjamin Franklin Memorial Hall. Approximately 350 persons attended the dinner and the presentation of awards. Prior to the dinner, a reception in honor of the 1951 Medalists was held in the rooms adjoining Memorial Hall.

The Medal Day Exercises opened with the playing of the National Anthem by Mr. Guy Marriner, Director of Music at The Franklin Institute. Mr. Richard T. Nalle, President, presided at the ceremonies.

The Stated Monthly Meeting of The Franklin Institute was held in conjunction with the Medal Day Exercises. The minutes of the Stated Meeting in May were approved as published in the June issue of the JOURNAL. Mr. Nalle announced that Sir James Chadwick, recipient of the Franklin Medal, had been elected to honorary membership in the Institute and that his certificate of membership would be presented to him *in absentia*.

The President then called upon Dr. John Frazer, Secretary of the Committee on Science and the Arts, to introduce the twenty-five former Medalists who were present.

Dr. Henry B. Allen was called upon to give the Secretary's report on "The Franklin Institute in 1951."

Following Dr. Allen's report, the presentation ceremonies for the fourteen medals awarded in 1951 were opened by a short message from the President, expressing regret that three of the Medalists were unable to be present to receive their medals: Sir James Chadwick, the Franklin Medal; Mr. Basil Albert Adams, a Potts Medal; and Mr. Eric Leighton Holmes, also a Potts Medal. Mr. A. K. Longair, Assistant Scientific Attaché of the British Embassy, accepted the medals for all three of these Medalists.

At the conclusion of the presentation of the awards, Mr. Nalle introduced Mr. Longair, who read the paper "The Beginnings of the Atomic Bomb," which had been prepared by Sir James Chadwick. This paper, as well as the full proceedings of Medal Day, will be published in the December issue of the JOURNAL.

HENRY B. ALLEN,
Secretary

PROGRAM

Receptation for Medalists	THE HOSTESS COMMITTEE
Assembly in Franklin Hall	Upon gong signal at seven-ten
National Anthem	GUY MARRINER
Dinner	MEDALISTS, OFFICIALS AND GUESTS
Toast to Benjamin Franklin	RICHARD T. NALLE, *President*
Greetings	THE PRESIDENT
October Stated Meeting of The Institute	THE PRESIDENT PRESIDING
Introduction of Former Medalists	JOHN FRAZER
The Institute in 1951	HENRY B. ALLEN, *Secretary*
Presentation of Awards	THE PRESIDENT
Presentation of Sir James Chadwick's paper on "The Beginnings of the Atomic Bomb"	

(Read by Mr. A. K. Longair, in the absence of Sir James.)

MEDALISTS

To be Awarded Certificates of Merit (1882)

JOHN F. McKEE
Research Engineer
Yarnall-Waring Company
Philadelphia, Pa.

JEAN MERCIER
President
Olaer Patent Company
Paris, France

To be Awarded a Longstreth Medal (1890)

HOWARD O. McMAHON
Physicist
Arthur D. Little, Inc.
Cambridge, Mass.

To be Awarded Potts Medals (1906)

BASIL ALBERT ADAMS
Research Chemist
Barry
Glamorgan, Wales

CLIFFORD M. FOUST
Electrical Engineer
General Electric Company
Schenectady, N. Y.

ERIC LEIGHTON HOLMES
Consulting and Research Chemist
Wollaston
Northamptonshire, England

To be Awarded a Levy Medal (1923)

ALBERT CHARLES WALKER
Research Engineer
Bell Telephone Laboratories
New York, N. Y.

To be Awarded a Henderson Medal (1924)

HERMANN LEMP
Consulting Engineer
Ridgewood, N. J.

To be Awarded Wetherill Medals (1925)

SAMUEL C. COLLINS
Professor of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Mass.

REID BERRY GRAY
Consulting Engineer
Baltimore, Md.

GAYLORD W. PENNEY
George Westinghouse Professor of Engineering
Carnegie Institute of Technology
Pittsburgh, Pa.

To be Awarded a Brown Medal (1938)

SAMUEL ARNOLD GREELEY
Sanitary Engineer
Chicago, Ill.

To be Awarded a Clamer Medal (1944)

JOHN CHIPMAN
Professor of Metallurgy
Massachusetts Institute of Technology
Cambridge, Mass.

To be Awarded the Franklin Medal (1914) and a Certificate of Honorary Membership

SIR JAMES CHADWICK
Master of Gonville and Caius College
Cambridge University
England

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

COULSON, THOMAS: The Centenary of the Foucault Pendulum.

GEORGE, E. F.: Light—Inadequacy of the Ultraviolet Theory of Ionization in the E-Layer.

ARCHIBALD, FRANK R.: The Step Shaped Film Applied to a Journal Bearing.

BUSH, VANNEVAR AND J. E. JACKSON: Correction of Spherical Error of a Pendulum.

GOLOMB, MICHAEL AND EUGENE USDIN: A Theory of Multidimensional Servomechanisms.

AYRE, ROBERT S.: Transient Vibration of Linear Multi-Degree-of-Freedom Systems by the Phase-Plane Method.

MCCLESKY, CHARLES S., JR.: Resistance of Plates with Small Electrodes.

NORINDER, HARALD: Investigation of Resistance and Power in Experimental Lightning Discharges.

MANDEVILLE, C. E. AND E. SHAPIRO: Radiations from Radioactive Rhodium (105).

SMITH, J. J. AND P. L. ALGER: The Use of the Null-Unit Function in Generalized Integration.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

It has seemed desirable for some time to have a formal statement setting forth the aims and objectives of the research policy of The Franklin Institute—particularly of the more recently organized Laboratories for Research and Development. To this end the Board of Managers adopted the Policy which is here published for the benefit of all concerned.

HENRY B. ALLEN
Executive Vice President

POLICIES RELATING TO RESEARCH AT THE FRANKLIN INSTITUTE

The general objective of the Laboratories of The Franklin Institute, as of all its other departments, is indicated in the full name under which it was chartered in 1824—The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts. It should be realized, of course, that under modern conditions the broader term "Physical and Engineering Science" is more appropriate than the narrower "Mechanic Arts" of earlier days.

Section 3 of the Institute's charter provides:

> The object of the said corporation shall be the promotion and encouragement of manufactures and the mechanical and useful arts, by the establishment of lectures on the sciences connected with them, by the formation of cabinets of models, minerals, machines, materials and products, by exhibitions and premiums, by a library and by all such measures as they may judge expedient.

As one of these other measures judged expedient for carrying out the provisions of the charter, the Institute has established Laboratories for Research and Development, where research is carried on under the sponsorship of private industry and government agencies; under grants-in-aid, fellowships, and special endowment funds; or under its own sponsorship when desirable and possible. In all such research work the Institute must, of course, retain full administrative control and responsibility.

The field of research, the physical and engineering sciences, is primarily organized to meet the needs of private industry. It embraces the development of basic knowledge and its application to useful products for the benefit of mankind. It is not the desire to carry on work of a conventional testing nature, although such work may be accepted if the Institute has unique facilities or equipment not readily available elsewhere.

It is the further policy and objective of the Institute to the extent possible to furnish and encourage technical education for its staff members, and to cooperate with students in our neighbor institutions and with research personnel from industrial organizations.

It is the policy of the Institute insofar as practicable to render research service to the City of Philadelphia, the State of Pennsylvania, and the Federal Government when such service is not inconsistent with its basic field of research or policies of operation.

It is the policy of the Institute to make its research facilities available to meet in the most practical way any national emergency. With the emergency passed, however, preference must be given to research of a long-term nature

involving fundamental knowledge in which the pressure of time and secrecy are not paramount and which can be undertaken by the staff with benefit.

The specific fields in which research projects will be undertaken, as well as specific projects in these fields, will be determined by the Executive Vice President of the Institute with the advice of the Executive Director of the Laboratories and his staff and of the Committee on Research of the Institute. Decisions are based on the type of work proposed, which must fit in with current policy and availability of space, facilities and personnel.

Since the promotion of useful knowledge is to a great extent brought about by the publication of the results of research, the publication of technical papers by the staff is encouraged whenever possible.

As to inventions, it is the policy of the Institute, where possible, to reserve a fair return to the inventor and to use any return it receives itself for the enlargement of the scope of its activities for the public benefit. The Institute currently has an agreement covering the development of patents with the Research Corporation, a non-profit organization which handles patents for the general benefit of all concerned and for the general advancement of science. In the case of research sponsored by government agencies or industrial organizations, employees will turn over to the sponsor inventions developed with sponsored funds in the specific subject field of the contract.

In addition to its Laboratories for Research and Development, the Institute also operates the Bartol Research Foundation which came into being as a result of the bequest of the residuary estate of Henry W. Bartol, left for the founding and maintenance of a research institute with preference to "researches in electrical science." Today electrical science comprises in its broad aspect a study of the basic fundamentals of physics, and the efforts of the Foundation have been directed mainly to this end. The close cooperation between the Laboratories for Research and Development and the Bartol Foundation provides for well-rounded research in the Institute as a whole.

All sponsored research in the Laboratories will be handled under contracts executed by the Executive Vice President of the Institute or other appropriate officials designated by the Board of Managers. Contracts will provide for periodic reports as desired by the sponsor, and it is expected that payments will be made monthly on the basis of work actually performed. Contracts will be on a basis of cost including a proportionate part of the Institute's overhead as well as such charges as may be necessary to maintain the equipment and facilities.

COLD ROOM EQUIPPED FOR SPECIAL TESTS

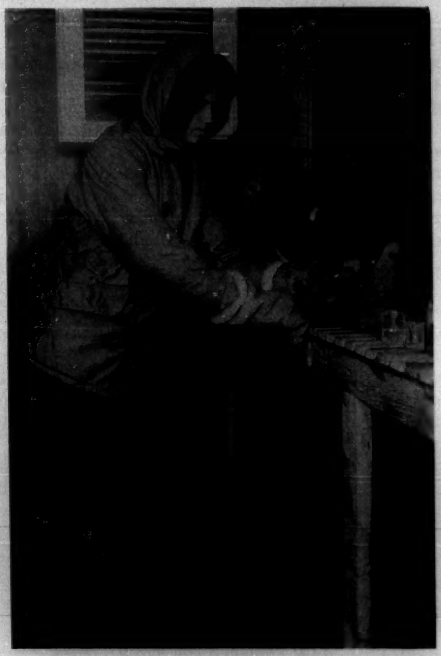
Another improvement has been added to the facilities of the Laboratories by the installation of a Cold Room in the Annex. It is 9 ft. × 7 ft., 8 ft. high, with an ante-room 5½ ft. × 8 ft.

While immediate need for the box is to run tests for the National Asphalt Research Center, it is an item of general utility equipment which is available for all laboratory projects having need for low temperatures—from −20° F. up to about 60° F. Asphalt work is of two types: 1) acquiring fundamental data which lead to new conceptions regarding the flow properties of asphalt, and 2) the running of performance tests which correlate with specific application requirements.

A recording thermometer permits one to tell what temperature is in the box and the reading for a week at a time. Minus 20° F. is selected as the coldest temperature which is of any commercial importance in the United States. Experience with many materials, partic-

ularly of a plastic or viscous nature, has shown that one cannot predict their behavior at low temperatures from tests run at room temperatures. For that reason, the use of the box will extend our knowledge of materials in a way that would be impossible without it. When high humidity air at room temperature is suddenly cooled, much of the water contained in it condenses and forms fog. Air at −20° F. contains very little water, hence humidity control at low temperature is not required.

Anyone using the Cold Room for low temperature tests should wear the special clothing provided for the purpose (Arctic boots, pants, parkas, and gloves), such as are shown in the



Clothing to be worn while making Cold Room tests.

photograph. It is possible to work an indefinite length of time at −20° F. if proper clothing is worn, otherwise one is limited as to the time he can work.

An inter-com system has been installed, so that anyone on the inside has only to speak and he can be heard outside.

Cold Boxes (as distinguished from the cold room) also are available in the Laboratories for special purposes, and one of them, a high altitude test chamber, can be operated as low as −80° F. In using such boxes for testing purposes, the test equipment is operated by remote control from outside.

MUSEUM

Ever since the famous fraud of Lemoine in the supposed growth of diamonds from sugar and the more realistic cultivation of pearls there has been a growing interest in the development of artificial jewels. Millions of gems are used industrially. Two thousands of them go into the instruments of a heavy bomber, some twenty millions are required annually for electric meters, and watchmakers take their share. There is, therefore, an obvious market for an artificial product of the same property at a reduced cost for these precious fragments of stone.

Synthetic emeralds were made in Germany before the war, but they were tiny. Today, emeralds are "grown" in San Francisco which measure forty carats, and there is talk of producing them at 200 carats—which is about the size of a golf ball. Rubies and sapphires are grown in laboratory furnaces in East Chicago at the rate of two feet a day.

The Museum has recently acquired an exhibit of a new kind of gem, the Kenya, which by actual test is more brilliant than a diamond. The Kenya Gem is not a synthetic, it is a jewel in its own right, crystallized by science and not by nature. It takes its name from Kenya Colony, the source of ilmenite or iron titanate, the basic ore. This is reduced to titanium, and subsequently converted into rutile or titanium oxide. The rutile is subjected to intense heat (3000 F.) to produce crystals which can be cut like diamonds. The crystals are tetragonal, the melting point is 3300 F., and hardness on the Moh scale is 7.

Relatively few diamonds are pure white, the great majority being tinged with color varying from brown to canary. The Kenya Gem resembles in shade the famous Florentine diamond which was in the Austrian Crown jewels. Because of its unique radiancy and brilliancy the Kenya Gem has found favor with jewelers (its refractive properties are 2.62 as against 2.42 for the diamond), but as it does not compete with the diamond in hardness it has not yet found industrial applications.

The exhibit in the Museum has been made possible through the co-operation of the Kenya Gem Corporation, Philadelphia.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, OCTOBER 17, 1951

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Lt. Col. Wm. H. Hollingsworth
Byron King
Martin W. Krause
Hrair A. Levonian
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Charles W. Berg '47
John S. Bioren '36
Dr. William H. Chandlee '45
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Charles Haydock '46
John Louis Osterlund '44

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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BOOK REVIEWS

HEAT PUMP APPLICATIONS, by Emory N. Kemler and Sabert Oglesby, Jr. 300 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $6.00.

The authors have capitalized on the growing demand for heat pumps which will both heat a dwelllng in the winter and cool it in the summer. Both have contributed much to the development of the heat pump through their inventions and through their published articles.

The book is primarily a summary of heat-pump literature published in both this country and Europe. There is an abundance of this literature available, and the authors have selected the material well and have included a lengthy bibliography at the end of almost every chapter. Many examples of the application of heat pumps to commercial, industrial, and residential installations are illustrated and described, usually accompanied by performance data.

The book takes up the various heat-pump cycles, explaining the advantages and disadvantages of each. Individual chapters treat the component parts of the heat pump, such as compressor, fan, electric motor, heat exchanger, and accessories. The thermodynamics of refrigerants within the heat pump is well treated. The chapter on heat sources is particularly good and includes information about many recent studies in this field. The reviewer feels that the psychrometrics, which is involved in air conditioning, has been slighted. A short chapter considers standard design data and comfort without reference to a psychrometric chart. A longer chapter discusses how to select an electric motor to drive the heat pump, one which would be quite suitable for a textbook in electrical engineering. A book treating heat pumps should place much emphasis on heat, and the study of psychrometrics and precise design data forms an important part of this thermodynamics.

The book includes many illustrative problems which are completely worked out. It does not include suggested problems which could be assigned to students for practice.

A. D. HAY

HYDRAULIC TRANSIENTS, by George R. Rich. 260 pages, diagrams and tables, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1951. Price, $7.00.

In his book, "Hydraulic Transients," G. R. Rich covers transient phenomena which may occur in open or closed hydraulic systems. Water hammer in hydroelectric power systems and surges in power canals comprise the principal items under discussion.

Beginning with a derivation of the basic equations encountered in water hammer, the author extends the theory to include the effect of various gate opening characteristics. He explains the use of Allievi charts and solves a variety of problems of a practical nature. The analysis includes the effect of water hammer upon the speed regulation of water turbines.

Next, systems for governing turbine speed are described and analyzed. The equations for these systems are derived and arithmetic integration is carried out for typical differential equations. Charts are included to facilitate the numerical method of solution.

The author covers the transient conditions which occur in pump discharge lines. Here again, the use of numerical integration to solve practical problems is illustrated. In this connection, the design of several types of automatic surge suppressors is described.

The author discusses the economic justification for the use of surge tanks in penstocks of hydroelectric installations. A rather complete mathematical analysis of surge tanks is given. Starting with the most elementary versions, the author amplifies the theory to include refinements and several types of surge tanks. Considerable numerical computations have been included to illustrate these analyses and designs.

Also considered are the filling and emptying of navigation locks as a transient problem occurring in hydraulics. Filling and emptying systems are described, analyzed and computations are given for several types of lock filling systems.

A chapter on surges in open power canals and canals subjected to tides has been included in the book. Sets of practical calculations are given to illustrate the procedure of computation. The effects of such factors as channel-bottom shape and friction are also included.

In general, the author has written a book well suited to the practicing engineer. Numerous numerical computations of authentic field problems give the reader a better feel for the problems involved and for the practical realization of their solution.

R. H. MARVIN

A HISTORY OF THE THEORIES OF ÆTHER AND ELECTRICITY, by Sir Edmund Whittaker. 434 pages, 16 × 25 cm. London, Thomas Nelson & Sons, Ltd., 1951. Price, 32/6.

In 1910, during his tenure of the office of Royal Astronomer of Ireland, Sir Edmund Whittaker published his "History of the Theories of Æther and Electricity." This work brought the subject down to the period when the quantum-theory and relativity changed the whole aspect of the investigation and rendered a new approach to the problem imperative. Retired from the active duties of a professorial chair, Sir Edmund is devoting his leisure to a revision of the book or, to be more correct, to a re-writing of it.

The projected work is to be complete in two volumes of which this, the first, deals with the classical theories. Few physical theories held such a tenacious grip upon the scientific mind. Descartes may be credited with having first given adequate outline to the theory when he viewed space as a *plenum*, being occupied by a medium which, though imperceptible to the senses, is capable of transmitting force, and exerting effects upon material bodies immersed in it. Thereafter, few physicists failed to accept the theory of the æther until the first decade of the twentieth century when it fell out of favor, largely as the result of the failure of the attempts to observe the earth's motion relative to the æther. But, Sir Edmund points out, the classical term may serve a useful purpose under the newer conditions. The substitution of the term "vacuous" for interplanetary space is not satisfactory, since the vacuum has come to be regarded as the seat of the "zero-point" oscillations of the electromagnetic field, and a "polarization" corresponding to a dielectric constant different from unity. It seems absurd, therefore, to retain the name "vacuum" for an entity so rich in physical properties.

The present volume, dealing with the classical conceptions of the æther, has been greatly improved by revision. It reflects a profound knowledge of the subject and leaves little further to be said. No science student should remain ignorant of the work performed by the mathematical electricians of the middle of the nineteenth century, and we can think of no more comprehensive study or better balanced judgment than that given in this work. Since the study ends with the classical theory in the age of Lorentz, we shall await with interest the completion of a work which promises to be the most authoritative study of the subject.

T. C.

VIBRATION AND SHOCK ISOLATION, by Charles E. Crede. 328 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $6.50.

As stated by the author, this book was written primarily to provide the practicing engineer with a comprehensive, yet practical, reference book on the design and application of resilient mountings. With this in mind, the author has produced a convenient reference book on the protection of equipment by the use of resilient supports.

The theory presented on vibratory systems, and its relationship to the design and application to vibration, shock or sound isolation, as reviewed by the author, is an excellent recapitulation of the old and new methods of handling such problems mathematically.

This book will be very helpful to designers who have not the desire nor the time to spend searching through textbooks on vibration analysis for helpful solutions to problems on equipment protections. This book concentrates on principles of isolation, on the design of isolators for several types of machinery, and on the damping properties of materials used in isolators, rather than on general vibration theory and practice. The author not only has covered his chosen subjects adequately, but he has included with each chapter a very comprehensive bibliography of books and papers relating to the subject.

HARLEY L. KRESSIN

INTRODUCTORY SOIL MECHANICS AND FOUNDATIONS, by George B. Sowers and George F. Sowers. 284 pages, diagrams and illustrations, 16 × 24 cm. New York, The Macmillan Company, 1951. Price, $4.75.

Before any large building, bridge, or dam is erected, the soil on which the structure will rest should be thoroughly analyzed for such engineering properties as permeability, compressibility and shear strength. The study of these and related soil properties is known as soil mechanics. It is a recent science and is still in the development stage. "Introductory Soil Mechanics and Foundations" adds another book to the growing list of volumes on this important subject of civil engineering.

As the title suggests, the book is for undergraduate study. It explains the various theories and applies them to practical engineering problems. Following an introductory chapter on the nature of soils including composition, grain size and shape and structure, the authors (G. B. Sowers, Lt. Col., Corps of Engineers, AUS (Res.) and G. F. Sowers, Associate Professor of Civil Engineering, Georgia Institute of Technology) discuss the physical properties of soils. What surface tension, shrinkage, permeability, compressibility and settlement, shear strength and rupture are, and how they affect the study of soils are all explained. A general chapter on soils and soil deposits follows. Here the various soils are described and identified. Seepage, drainage and frost action are next explained including means of controlling ground water flow. Types and requisites of good foundations—footings, mats and piles—are described. The description of the theory of earth pressure and its practical application to the design of retaining walls and excavation; of the construction of fills and subgrades including compaction and stabilization; and of the stability of earth masses complete the specific subjects in the volume.

The concluding chapter on underground investigation might well be read initially, since it outlines the details and purpose of a soil investigation. It would be well for every contractor to read this chapter. There are many examples in the history of construction where contractors have come to grief because they failed to make an adequate subsurface study before commencing to build. The surface may look all right but an old dump may be underneath, or soft clay; or unusual groundwater conditions may exist that would be ruinous to any supersurface structure. While not original, which the author freely admits, the book affords a sound introduction into the new but steadily expanding field of soil mechanics.

E. W. HAMMER, JR.

DATING THE PAST, by Frederick E. Zeuner. Second edition, 474 pages, plates, charts and diagrams, 15 × 22 cm. London, Methuen & Co., Ltd., 1950; distributed in U. S. by Longmans, Green & Co., Inc., N. Y. Price, $5.50.

One of the most fascinating of studies is that which tries to reconstruct a geological history of the earth. This subject of geochronology aims for the establishment of an absolute time scale for the past and to do this fields like geology, botany, zoology, astronomy, and physics are made to contribute in hope of overlapping to provide the absolute measure called for.

In "Dating the Past," Dr. Frederick E. Zeuner, professor of environmental archeology in the University of London has brought up-to-date the first edition of his book first published in 1946.

The book is divided into four sections. Dating back 3000 years to 1000 B.C. is based on the ring analysis so brilliantly conceived by Dr. A. E. Douglas in 1901. This subject of dendrochronology comprises section I.

Part II goes back some 15,000 years and is based on varied clays.

Part III deals with the chronology of the Pleistocene going back about a million years and Part IV deals with the least known chronologies and goes back some three billion years to the beginning of the earth.

This reviewer has long been intrigued by clever step-by-step correlations of the rings to give an unbroken history going back some 3000 years. He considers it a tribute to the keen insight of the dendrochronologists that they can so completely invest the climatic past through tree ring analysis. Their work almost reads like a detective story in the building up of the records clue by clue.

Part III is of particular interest to this reviewer because of the astronomical data used to unravel the history of the ice ages. While he does not agree with the astronomical chronology of the Pleistocene based on various periodicities of the earth, the theories explored to account for the fluctuations in climate are most absorbing.

"Dating the Past" is a treasure trove of information for the geologist, astronomer, biologist, meteorologist and others in the allied fields. It is one of the most important reference books today.

I. M. LEVITT

TRANSIENTS IN POWER SYSTEMS, by Harold A. Peterson. 361 pages, diagrams and photographs 16 × 24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $6.50.

The main emphasis in this book is on the analysis of situations which produce overvoltage in electric transmission facilities and on the derivation of expressions for computing the magnitude of the overvoltage. Preventive measures and their theory of operation are also considered. The effects of switching, arcing, open conductors, nonlinear elements and other causes of transient phenomena are studied. For these cases, system characteristics are determined analytically or, in some instances, by the use of a miniature system and transient analyzer.

The treatment of transient phenomena in this volume, an extension of the pioneering work of men such as Steinmetz and Heaviside, makes use of more recent tools like the differential analyzer to derive solutions for complex problems. In this the author proceeds at a very gradual pace, and all of the concepts presented are easily understood by engineers with average mathematical aptitude.

The equations are developed by the use of the generalized method of symmetrical components and the substitution of operational impedance expressions. Usually the resulting derivation is plotted for a variety of values of the controlling parameters so that the significance of the equations is clearly illustrated. To further emphasize the validity of these equations a comparison with test results is often included in the form of photographic oscillograms taken from operating apparatus.

Since the subject of this book is transients in power systems, the author could have added to the scope of his book by discussing at greater length network analyzers, of which there are twenty or more in the United States, as well as their capabilities and limitations. The great expansion of electric generating utilities in the past decade has made network problems increasingly complex and requires more and more the use of the network analyzer for solutions. Although brief mention is made of the differential analyzer and several results obtained with the transient analyzer are presented, more could have been explained about their use.

Professor Peterson's book will be well received by persons concerned with the design of power systems and transmission apparatus, for it contains the elements of practice as well as many useful analytical results. It will also be useful as a text for electrical engineering power majors. In addition to the conventional author index, an interesting bibliography by dates is provided starting with 1900 and containing all important developments in the power systems field to the present.

C. W. HARGENS

SOIL TESTING FOR ENGINEERS, by T. William Lambe. 165 pages, illustrations, 22 × 29 cm. New York, John Wiley & Sons, Inc., 1951. Price, $5.00.

While several books on the theory and practice of soil mechanics have appeared in recent years, none has been exclusively concerned with the methods of soil testing. "Soil Testing for Engineers," by T. W. Lambe of the Massachusetts Institute of Technology, fulfills this gap in the literature and provides a ready reference for the common tests used in soil mechanics.

Except for the introduction, the book is solely concerned with descriptions of tests. In the introduction the author discusses soil identification and description, storage and handling of soil samples; and describes water content, void ratio, porosity and degree of saturation. The various tests described are for specific gravity, Atterberg limits and indices, grain size, compaction, permeability, capillary head, capillarity-permeability, consolidation, direct shear and triaxial compression on cohesion and cohesionless soil, and unconfined compression. A

chapter is assigned to each test and follows the same format: introduction, apparatus and supplies, recommended procedure, discussion of procedure, calculations, result and numerical example. Generally, an illustration of the tabulated form used in assembling the data is also included, as well as schematic diagrams and photographs of the test equipment. There are two appendices: one including conversion factors, specific gravity and viscosity of water, pictures and descriptions of proving rings and liquid limit devices; and the other, derivations of equations used in the text.

Although short and concise, "Soil Testing for Engineers" contains an unusually complete description of the various soil mechanics tests. It should prove a very useful manual for the student or practicing engineer.

E. W. HAMMER, JR.

FUNDAMENTALS OF ELECTRICAL ENGINEERING; by Fred H. Pumphrey. 668 pages, illustrations, 14 × 22 cm. New York, Prentice-Hall, Inc., 1951. Price, $5.75.

This book deals with the fundamentals of electrical engineering and their application in practice. It is intended as a text for students who are specializing in fields other than electrical engineering. Approximately two thirds of the book is devoted to a limited presentation of direct and alternating current circuits, measurements, and machinery. It is felt that the major contribution to pedagogy in this section is the introduction of clear photographs of modern equipment; the text material is not, in many cases, comparable in depth to that found in other textbooks intended for a non-electrical engineering student.

The last third of the book is devoted to elementary electronic techniques. Sufficient detail is here presented for the student to design simple amplifiers and control devices.

S. CHARP

A CONCISE HISTORY OF ASTRONOMY, by Peter Doig. 320 pages, 14 × 22 cm. New York, Philosophical Library, 1951. Price, $4.75.

It seems odd that the revolutionary advances in the field of astronomy in the past few decades no comprehensive work has appeared to show the progress of this century.

This has been remedied by the appearance of "A Concise History of Astronomy" by Peter Doig, editor of the *Journal of the British Astronomical Association*.

Of the 320 pages roughly 200 are devoted to the history of astronomy up to the 20th century. This has been covered many times by many people like Clerke, Bryant and Forbes. The merit of this book does not lie in this section—rather its value is in its treatment of 20th century astronomy.

Mr. Doig has devoted more than a third of his book to the findings of the 20th century. It was Fred Hoyle who said, "Just as a blazing bonfire is to a penny candle, so is the observational progress achieved in the last few decades to the work that came before it." In these 100 pages Mr. Doig has presented an almost complete development up to the present date. Instruments are discussed from the spectroheliosope to radar and the coronograph. A detailed resumé of our knowledge of the sun and solar system is given in a lucid, compelling manner. The tremendous advances in stellar astronomy are described and the intense work on stellar interiors fully discussed.

Finally, Mr. Doig deals with the most fascinating aspect of Astronomy—the velocity-distance relation for the galaxies.

To this reviewer the lack of "histories" in so long a time is regrettable but this history was worth waiting for.

I. M. LEVITT

THE EARTH'S MAGNETISM, by Sidney Chapman. 127 pages, illustrations, 11 × 17 cm. London, Methuen & Co., Ltd., 1951; distributed in U. S. by John Wiley & Sons, Inc. Price, $1.50.

This is a revised edition of a book first published in 1936, well known as the only easily available introductory book on the subject. The changes in this edition are minor, but all of such character as to add to the book's value, since they incorporate the results of the more recent investigations.

The number of chapters and their subject matter have not been changed, and only small additions to the text have been made, principally a discussion of the anomalous daily variation at Huancayo Observatory. Figures have been revised and increased in number; the isomagnetic charts now include vertical intensity, inclination and total intensity as well as declination and horizontal intensity, and of course are drawn from the later date of 1945. The isoporic chart has been revised to show the epoch of 1940–1945 and is now a micator projection. Figures (from Chapman and Bartels, *Geomagnetism*) have been added to display the system of ionospheric currents which would account for the externally produced portion of the disturbance field.

The text is brief, as it must be to be presented in 116 pages. It suggests, rather than details, the painstaking analyses which are required to separate the many phenomena which enter into a description of the earth's magnetic field and its variations.

ALAN C. BYERS

PLANT LAYOUT. DEVELOPING AND IMPROVING MANUFACTURING PLANTS, by John A. Shubin and Huxley Madehaim. 433 pages, illustrations, plate, 15 × 24 cm. New York, Prentice-Hall, Inc., 1951. Price, $5.50.

Students of industrial engineering and management and business executives who are concerned with industrial plant facilities are the intended readers of this new textbook. The subject matter moves from the tangible to the intangible, always keeping in mind that the primary means of improving the efficiency of production operations are good product design and efficient layout of plant facilities. Selecting the more significant essentials of good management from the less is indeed a difficult task. The authors have succeeded, however, in presenting important principles, modern techniques and procedures connected with the selection and layout of plant facilities which lead to improved operations.

The scope of the subject matter is wide indeed. Following a presentation of the principles of the survey which should precede the establishment of an enterprise, the reader is made acquainted with the relationship between product design, selection of equipment, and industrial processes. All three must be considered carefully prior to the formulation of major decisions concerning new products or inefficiency and/or failure of operations may result.

Plant layout influences production, inspection, maintenance, product design, planning and control, purchasing, and personnel management. It should, therefore, be an over-all top management function and to be considered in the choice of the geographical location of a plant, in the establishment of internal operations and *modus operandi*, and process and product moving inside the plant. These topics are treated by first introducing the principles and then in many cases following up with specific cases for illustration. Frequent use is made of charts, photographs and tables.

The efficient layout of new plants and the revising and improving of existing plant layouts to achieve the objectives enumerated in the earlier sections of the book are considered in the later sections. Numerous examples are presented to illustrate the problems which may be encountered in a variety of manufacturing plants.

Although the authors have achieved the general objectives of their work, it is felt that more emphasis should have been placed upon those economic factors which may affect decisions regarding the purchase and long-term use of plant and machinery. Such factors often counteract the principles of efficient plant layout.

S. CHARP

THE PREPARATION OF PROGRAMS FOR AN ELECTRONIC DIGITAL COMPUTER, by Maurice V. Wilkes, David J. Wheeler and Stanley Gill. 167 pages, illustrations, 15 × 23 cm. Cambridge, Addison-Wesley Press, Inc., 1951. Price, $5.00.

The preparation of programs for digital computers is a very important step in the utilization of these computers. It appears that these machines will be used in ever expanding numbers as their usefulness and efficiency become apparent to smaller organizations. Therefore, to facilitate their use, it is important that the necessary preparations for their use be simplified as much as possible. Many subroutines will occur over and over again, sometimes with slight

modifications, often intact. A library of such subroutines and facilities for using it would be of great assistance in the preparation of programs. It would not only reduce the labor of programming calculations in detail, but would also reduce the time of both the programmer and the machine in finding and correcting mistakes in the program.

The authors, who have been primarily concerned with setting up such a library of subroutines at the Mathematical Laboratory of Cambridge University, have gathered their material for this book from a considerable amount of exploratory work on such matters as ways in which to specify operating instructions to the machines, how to set up subroutines so as to give the required flexibility, and how to make use of these subroutines in the computer. The book adds much to the available knowledge on the techniques of effectively using subroutines in an automatic calculating machine.

Although the system of subroutines discussed and given in this book might not be applicable to all machines, it can serve as a pattern which will greatly facilitate the development of such a system for a particular machine. The authors include many subroutines which not only are very useful, but also show the great flexibility of the electronic digital computer. One chapter is devoted to examples of possible and actual problems prepared with this system.

This book is a good treatment of the preparation of programs for automatic calculating machines. Since this subject is still being developed, the book should help others working in this same field. Certainly, in the future, others will have much to add as the field is further investigated.

GEORGE O. PETERS

UNIVERSAL MECHANICS AND HAMILTON'S QUATERNIONS, by Otto F. Fischer. 356 pages, 14 × 21 cm. Lidingö (Sweden), Axion Institute, 1951. Price, $10.00.

The author, a consulting civil engineer and founder of the Axion Institute, published this book in lithoprint from a typewritten manuscript as "an attempt to introduce a certain order in the mathematical structure of universal mechanics by means of the calculus of Hamilton's quaternions." The subtitle, "A Cavalcade", serves to indicate "a rather erratic path of approach . . ., a wandering together with the reader through many 'countries' . . . in which the author has found new beauties and exhilarating outlooks."

Having found quaterions useful in the solution of certain complex problems in elasticity (which constitutes the "Axion method") and having published several papers on their application in other fields, the author "has made up his mind, for want of other, more authoritative, works to give many aspects of the possibilities of Hamilton's quaternions" in the present book.

A 34-page review of Gibbs' vector analysis serves to introduce the reader to concepts which find their generalizations in simple, quadric, and double-quadric quaternions, to the exposition of which the principal part of the book (265 pages) is devoted. The last 40 pages contain a review of further developments and ideas relating to the problem of "universal mechanics" conceived as an all-comprehensive unified mathematical theory of the physical world embracing cosmology, "molar physics" and "micro-physics," including quantum mechanics.

In view of the wide scope of the subject, the material is presented in a very concentrated form. The definitions of simple and double quaternions and quaternionian operators are introduced using specific amplifications in the order of increasing complexity found in the existing literature, collected in an appended bibliography with 124 entries.

Beginning with quaternion representation of rigid rotations and of affine deformations in three-dimensional space, the author proceeds rapidly to Minkowski's 4-space, non-Euclidean geometry, the restricted theory of relativity, the energy-momentum tensor, the Riemann-Christoffel curvature tensor, Einstein's theory of gravitation. He gives a brief exposition of the formalism, adopted by the various workers in this field, and points out the equivalence or discrepancy, as the case may be, between their quantities and operators and those of the quaternion calculus, in part, as developed further by the author himself.

A similar, but even briefer, exposition is given of Eddington's E-numbers and Dirac's matrices which are shown to be of the same order as quadric quaternions. Lastly, Küssner's

use of quaternions in his *Principia Physica* (1948), is discussed in some detail as representing the closest existing approach to the author's conception of a "potential pyramid"—a hierarchy of potentials derivable from an ultimate potential by repeated differentiations and analogous generalized quaternionian operations.

Throughout the book the problem of electro-mechanical parallelism is discussed at various stages of the exposition. The author scorns the "superficial electro-mechanical analogies" familiar to engineers, pointing out the practical advantages of more subtle analogies that would be provided by a unified theory exhibiting this parallelism.

The author concludes: "The book is full of loose ends, hardly started yet, and open to development by better investigators."

The present reviewer wonders if this book will fulfill its intended purpose of arousing wider interest in engineering applications of the quaternion calculus. He feels that a judicious selection from the available material presented in a less condensed and more highly organized form would have had more "reader appeal."

A very serious drawback is the consistent use of faulty English, which may render portions of the text partly or fully unintelligible to the English-speaking readers. Particularly misleading is the use of "by" as an equivalent of the German "bei" (or "dabei"), and of "on" instead of "in" in numerous instances.

Even though the present reviewer does not share the author's expectations of a quaternionian panacea in an ultimate unified theory of the physical universe, he feels that a reader who is not deterred by the above obstacles will be well rewarded by a deeper insight into the theory of quaternions and of "geometrical objects" in general.

OTTO R. SPIES

BOOK NOTES

PRINCIPLES OF ELECTRICAL ENGINEERING, by William H. Timbie and Vannevar Bush. Fourth edition, 626 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $6.50.

The authors have presented in this fourth edition of their text material suitable for the understanding of methods of electrical engineering. They have stressed methods for analyzing electric and magnetic problems, circuit analysis, fundamental problems of the electric and magnetic fields and their relation to circuit analysis. The material is based on mathematical physics as applied to the particular field.

GALVANIZING (HOT-DIP), by Heinz Bablik, translated by Clive A. Bentley. Third edition, 502 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $10.00.

The author has presented in this third edition a technical treatise covering both the theory and practice of hot galvanizing. Discussions based on the practical and theoretical aspects of scale, pickling, fluxes and galvanizing are covered. The book leans heavily on the theoretical, is well illustrated, and the sections on practice would be of use to any one in the industry.

PLANE TRIGONOMETRY, by Lyman M. Kells, Willis F. Kern and James R. Bland. Third edition, 220 pages plus 118 pages of log tables, 15 × 23 cm. New York, McGraw-Hill Book Co., 1951. Price, $3.50.

In this third edition of a text first published in 1935, practically every section of the book has been revised. Changes have been made in the order of presentation of material to improve the teaching of the subject; graphs have been given greater emphasis; a chapter on vectors has been added; some problems have been rewritten, others eliminated and new ones included; and forms for computation include not only the natural forms but also concise ones, which provide the student with short cuts. The text in its revised form should be easy to teach and understand. The five-place log tables (118 pages) are those from the 1935 edition.

ELEMENTARY PROBLEMS IN ENGINEERING, by H. W. Leach and George C. Beakley. 269 pages, diagrams and illustrations, 16 × 24 cm. New York, The Macmillan Company, 1951. Price, $3.50.

Since this text was written primarily for engineering problems courses, the material is of freshman level. The problems are simple and include use of the slide rule, logarithms, elementary trigonometry, equations of motion and the basic principles of mechanics. Two introductory chapters give the student a preview of what is expected of him in engineering; subsequent chapters show him, through very clear examples, the types of problems he must solve as an engineer. Drill problems accompany each chapter, and the many illustrations and tables add greatly to the presentation of material. The simple style and the variety of problems make this a good freshman text.

MATHEMATICS FOR ENGINEERS, by Raymond Dull. Third edition revised and edited by Richard Dull, 822 pages, diagrams, 15 × 21 cm. New York, McGraw-Hill Book Co., 1951. Price, $7.50.

There is no need to do more than note the changes in the third edition of this well-known reference book for engineers. New chapters have been added on differential equations and dimensional analysis, with special emphasis given to the Buckingham Pi theorem and its applications to engineering analysis. The book will continue to aid the engineer who wants to review the mathematics necessary for the practice of his profession.

IRRIGATION ENGINEERING, VOL. 1, by Ivan E. Houk. 545 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $9.00.

Volume 1 of *Irrigation Engineering* is devoted to agricultural and hydrological phases of irrigation. Emphasis is on the engineering aspects, with summaries of field and laboratory investigations. Included among the subjects discussed are: soils, soil-moisture movement; land evaporation; irrigation of saline and alkali soils; runoff and streamflow; losses and waste of irrigation water. This work brings up-to-date a science that has developed greatly in the last quarter century and, with Vol. 2 on project planning and structures, will be one of the most complete treatises on modern irrigation engineering.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

***Abstract of* Highly Polymerized Ribonucleic Acid: Preparation from Liver and Depolymerization.**—E. L. GRINNAN AND W. A. MOSHER (*J. Biol. Chem.*, **191**, 719 (1951)). Highly polymerized ribonucleic acid has been prepared from calf and rat liver. The procedure involves the maintenance of the pH between 6 and 7, the precipitation of a nucleic acid–protein complex from the cytoplasmic extract according to the method of Volkin and Carter, heat denaturation of the protein for 2 minutes at 100°, and removal of the protein by shaking with chloroform.

The average apparent molecular weight of the RNA of a typical preparation, calculated from the sedimentation-velocity constant, was 264,000, corrected for viscosity of the solvent, and 580,000, corrected for viscosity of the solution. The axial ratio was 70:1, and the intrinsic viscosity was 319. Anomalous viscosity and moderate streaming birefringence were exhibited by the RNA solutions.

The RNA was depolymerized by x-radiation, chemically generated free radicals, and sonic vibration.

***Abstract of* Effects of a Protein-free Diet on the Alkaline and Acid Phosphatase Activity of the Liver of the Rat.**—J. O. ELY AND M. H. ROSS (*Nature*, **168**, 323 (1951)). Alkaline phosphatase has been shown chemically and cytochemically to be increased in both nucleus and cytoplasm during protein depletion produced in rats by the feeding of a protein-free diet. Acid phosphatase, on the other hand, has been shown cytochemically to be reduced in both nucleus and cytoplasm. Other proteins of the liver are apparently reduced to a greater extent than acid phosphatase.

CURRENT TOPICS

Effect of Brain Surgery on Learning Process.—New facts concerning the role of the cortex of the brain in learning and memory have been announced by a University of Wisconsin psychologist.

The psychologist, Prof. Karl U. Smith, based the announcement on before-and-after psychological tests given brain surgery patients at Rochester, N. Y., and on studies of brain function and learning in animals. Professor Smith's report appears in a recent issue of *Science*.

The cerebral cortex, or outer covering, of the brain has hitherto been supposed to be the main organ functioning in learning and memory. The neurones, or nerve fibers connecting different areas of the brain, were, it was believed, the direct participants in the acts of learning and memory. It was also believed that the neurones were changed in some way when learning was accomplished, establishing certain connections and pathways in the cortex.

In Professor Smith's studies, the effect of serving these association neurones upon leanring and memory was investigated.

He also studied the effect of such operations upon the so-called bilateral transfer of learned response. This bilateral transfer of learning refers to the fact that if a task has been once learned with the right hand it becomes much more easily learned with the left hand. It has been thought that this transfer of learning from one side of the body to the other was also dependent upon the neurone connections between the two cerebral hemispheres.

Before-and-after testing of human patients and of animals has shown, however, that destruction of the various association neurone systems of the cerebral cortex has no significant effect upon the capacity to learn new habits, to remember learned habits, or to transfer habits from one side of the body to the other.

"Extensive destruction of most of, if not all, the neural connections between the two cerebral cortices produces either no diminution—or at least an inconsistent diminution—of the ability of human subjects to transfer to the opposite side of the body motor habits learned by use of the preferred side," Professor Smith says.

The conclusion drawn from the work is that the connecting nerve fiber system of the brain must be something other than an associative system for learning and memory.

"The various neural concepts now accepted to explain learning and memory are seen to have no meaning whatsoever. There must be some other—as yet unknown—mechanism involving different areas of the brain which accounts for the occurrance of memory and of learning," Professor Smith points out.

"Taken literally," he said, "the available experimental results would seem to stress especially the possibility that the cortex, in contrast to traditional thinking, is primarily a reaction system. . . . If these possibilities are found to be true in fact, the problem of analyzing the nature of learning changes within the brain may turn out to be far more difficult that it is now conceived to be."

Automatic Marine Pilot System.—A completely automatic marine pilot system which unerringly follows any course it receives from a human or automatic source has been developed for the U. S. Navy by engineers of the General Electric Company.

Where a helmsman frequently overcontrols the rudder in wind, swells and currents, the auto-pilot follows its course with an average variation of less than one-half of one degree.

Because the rudder is moved less often, some fuel savings should result from use of this equipment.

A second advantage, according to G. E. engineers, is the almost complete lack of maintenance required on the auto-pilot as compared with other automatic steering systems. Primary reason for this is the use of magnetic amplifiers, which have proved to be extremely capable of enduring the rigors of shipboard use, the engineers said. Amplifiers are used to step up a small amount of power to a higher power level when it is needed.

The G. E. auto-pilot has several operating highlights not found in manual systems. These include:

An automatic compensator which "trims" the ship when there is more force or resistance on one side of the vessel than the other. This keeps the vessel from taking a circular course when the majority of the power comes from one side.

A weather adjustment which may be set to minimize rudder action in heavy seas, thereby cutting down the stresses on the hull caused by excessive rudder movement.

An override which permits the helmsman to take the ship immediately from automatic control and place it under manual operation in emergencies.

An automatic turn compensator which keeps the ship from maneuvering past the desired compass heading in completing a turn. Frequently, helmsmen find it difficult to stop their vessels from turning past the desired point. This overcontrolling factor and resulting power waste have been eliminated with the auto-pilot.

The auto-pilot, now being installed on a Navy prototype destroyer, is capable of being used on any type vessel, G. E. engineers stated.

New Anesthetic Drug.—A search of several years for a better all-around anesthetic drug has ended in the development of Surital sodium at Parke, Davis & Company, Detroit, Mich.

Dr. E. A. Sharp, director of clinical investigation, said, "Surital is a highly-potent compound which has proved to be in some respects superior to any intravenous short-acting anesthetic agent now available."

Surital, a new thiobarbiturate, already has been found to be safe and effective on thousands of patients. Among its many advantages are high potency; smooth and rapid induction of sleep, free of undue excitement or irritability; early and easy recovery, with infrequent side reactions; minimum respiratory depression; applicability over a wide age range; and usefulness among infants and children because of its effectiveness rectally.

Surital is currently available only to certain hospitals and it will be several months before production allows general distribution.

The rectal use of Surital has proved to have far-reaching possibilities for anesthesia in operations about the head, nose and throat; in dental work; and for the use of instruments in examinations of the lungs, Dr. Sharp said.

According to specialists using the drug, it enables 100 per cent oxygen to be delivered to the patient throughout the procedure. It is but moderately depressant to respiration and produces a lower incidence of severe laryngo-spasm or broncho-spasm than other barbiturates in current usage. Also, induction of anesthesia is pleasant and provides amnesia so that repeated procedures are readily tolerated. Post operative sleep is of short duration, and protective reflexes return early. The explosion hazard (from ether) does not exist.

Dr. Sharp said patients selected for Surital anesthesia have represented every age group, a wide variety of operative procedures, all types of surgical and anesthetic risks and, in conjunction with other agents, cases in which anesthesia has been required for periods of three minutes to six hours.

Most patients are usually awake five to 10 minutes after the last injection and recover full rationality within 10 to 20 minutes.

The scientific designation of the new drug is sodium 5-allyl-5-(1-methyl-butyl)-2-thiobarbiturate, its initial pharmacologic study having been made by Dr. M. H. Seevers and his associates in the Department of Pharmacology at the University of Michigan.

Heating Cable Keeps Runways Free of Snow.—Concrete crane and transport runways are being kept clear of snow by means of a novel application of heating cable at the South Milwaukee plant of the Bucyrus-Erie Company.

The company lined the runways with General Electric flexible, lead-covered heating cable just before the worst part of the winter last year. The installation, comprising 40,000 feet of heating cable, covers four concrete runways ranging from 100 to 275 feet in length and about ten feet wide.

Step-down transformers, to supply correct voltage for the non-standard lengths of heating cable, were built especially for this installation by G. E.'s Milwaukee service shop. This required secondary voltage is about 180 volts for the shortest runway and about 500 volts for the longest.

The heating cable is laid on wire mats about 2½ to 3 in. below the surface and on approximately 3-in. centers. Tile drains installed along the edges of the runways provide adequate drainage for the melted snow.

According to Bucyrus-Erie officials, the heating cable met its greatest test to date late last winter after a weekend snowfall. Even with the outside temperature at eleven degrees below zero, the cable was able to clear the runways in a few hours.

Canned jet engines are expected to save the Air Force over $6 million yearly, through protecting engines from damage in shipping, and eliminating frequent inspections required to guard against rust and corrosion when engines were packed in wooden cases. The containers are made of low-carbon hot-rolled steel plate, utilize rolled flanges and rubber "O" ring for moisture and air seal. Engines are rubber-mounted inside the cans, can withstand total immersion in water or 36-in. drops with no damage.